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Free vibration analysis of micro rotating beams based on the strain gradient theory using the differential transform method: Timoshenko versus Euler-Bernoulli beam models

Hadi Arvin

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Abstract

In this paper, a novel formulation for the micro rotating beams based on the strain gradient theory and the assumptions of Timoshenko and Euler-Bernoulli beam models is developed. By introducing the prestressed configuration induced by the centrifugal forces, the obtained equations of motion are linearized about the prestressed configuration. Some dimensionless parameters are assigned to achieve the nondimensionalized equations of motion. Differential transform method is applied on the equations of motion and the associated boundary conditions to obtain the flapping and axial natural frequencies. The current outcomes are validated by those available in the literature. After validation of the obtained results, some case studies are prepared to define the material length scale, the rotation speed and the slenderness ratio influences on the flapping and axial frequencies. The outcomes illustrated the natural frequencies are very sensitive to the case study parameters. Also, employing the Euler-Bernoulli beam model leads to the incorrect results in calculation of the flapping frequencies for a wide range of slenderness ratio even when the material length scale is close to the micro beam thickness.

Introduction

Dealing with the micro and nano structures demands some revisions on the classical formulation by inclusion of the size effects. The theories where established for including the size effects in their proposed formulation have been known as nonclassical theories. The most exploited nonclassical theories are the nonlocal theory (Ganghoffer et al., 1999), the couple stress theory (Ottosen et al., 2000) and the strain gradient theory (Chen and Wang, 2001). This paper scope, is to develop a new formulation for modeling the rotating micro beams by including the size effects on the basis of the strain gradient theory (SGT).

Kong et al. (2009) derived an analytical solution for the static deformation and the natural frequencies of the Euler-Bernoulli micro beams which were modeled based on the SGT. The obtained results demonstrated the SGT causes the beam behaves stiffer and hence, a noticeable decrease in the static deflection and a significant increase in the natural frequencies is evident when the material length scale parameter is close to the micro beam thickness. Wang et al. (2010) obtained the equations of motion (EOMs) for a simply-supported Timoshenko micro beam on the basis of the SGT. The obtained results for the natural frequencies and the static deflection illustrated the noticeable role of the material length scale parameter when it is close to the micro beam thickness. Lazopoulos and Lazopoulos (2010) implemented the SGT with the surface energy effects to attain the Euler-Bernoulli micro beam EOMs. They investigated the buckling analysis and the effects of the cross sectional area in bending of the thin micro beams. On the basis of the SGT, Asghari et al. (2012) developed the EOMs of the Timoshenko micro beams which undergoes the large deformations. They examined the nonlinear static responses and the free vibration features according to the obtained nonlinear EOMs. Zhao et al. (2012) investigated the nonlinear static deflection, the critical bucking loads and the nonlinear natural frequencies of the Euler-Bernoulli micro beams founded on the SGT. They illustrated the importance of the material length scale parameter when it is comparable with the micro beam thickness and the determinative influences of the nonlinearity on the obtained results. Based on the SGT, Ramezani (2012) proposed the nonlinear formulation of a simply-supported Timoshenko micro beam. He demonstrated the hardening effects of both the geometric non-linearity and the SGT on the natural frequencies. The thermal influences on the nonlinear buckling examination of the Timoshenko micro beams was studied by Mohammadi and Mahzoon (2014). The presented formulation on the basis of the modified SGT predicted higher critical buckling load rather than the classical theory (CT) and the modified couple stress theory (MCST). Zhang et al. (2014) employed the third-order shear deformation theory to find the EOMs of a functionally graded (FG) micro beam rest on an elastic foundation founded on the SGT. They derived the analytical solution for defining the natural frequencies, the static deflection and the critical buckling load for the simply supported micro beams using the Navier method. They studied the material length scale parameter, the elastic foundation parameters and the shear deformation influences on the static and free vibration features of the FG microbeams. Belardinelli et al. (2014) examined the static deflection and the pull-in instability of a clamped-clamped microbeam which was modeled base on the SGT by employment of the differential quadrature method. The nonlinear midplane stretching and the fringing field influences were included in their presented formulation. The obtained results for the pull-in limit were compared by the outcomes on the basis of the CT. The numerical analysis demonstrated the larger static response near the pull-in instability region based on the SGT rather than the CT outcomes. A nonlinear dynamic study was also performed on the reduced order model obtained by the Galerkin discretization approach to show the importance of the nonlinear terms included by the SGT. Ansari et al. (2015) developed a new Timoshenko beam element included by the size effects based on the SGT to analyze the static deflection and the free vibration of the Timoshenko microbeams. Nojoumian and Salehieh (Nojoumian and Salarieh, 2016) discussed on the boundary conditions (BCs) of the Timoshenko micro beams proposed by Wang et al. (2010). They illustrated the presented BCs in Ref. Wang et al. (2010) are not capable to reduce to the classical BCs and they presented the new correct BCs which had this ability. Xu and Deng (2016) formulated the Euler-Bernoulli micro beams with higher-order inertia founded on the SGT. They proposed the analytical solutions for the different BCs and examined the material length scale parameters, Poisson's ratio and the BCs effects on the obtained natural frequency results.

Due to the pivotal role of the rotating structures for the designers and engineers, numerous researches addressed the vibration and stability examination of the rotating beams and blades. Arvin and Bakhtiari-Nejad (2011) applied the discretized method of multiple scales to the nonlinear coupled flapwise and axial EOMs of a rotating Euler-Bernoulli beam to achieve its nonlinear normal modes. They examined the stability of the nonlinear normal modes in the presence of the internal resonances. Lacarbonara et al. (2012) developed the geometrically exact formulation for the rotating blades based on the Cosserat theory of rods. They implemented the Galerkin discretization approach to the governing differential EOMs to obtain the flapping, lagging, axial and torsional natural frequencies. They investigated the influences of the rotational speed and the Coriolis forces on the achieved natural frequencies. Arvin and Bakhtiari-Nejad (2013) derived the nonlinear EOMs of a rotating composite Timoshenko beam. They examined the nonlinear free vibration analysis by implementing the direct method of multiple scales. A rotating composite Timoshenko beam was modeled by Georgiades et al. (2014) to derive the governing EOMs. The formulation was included by the nonclassical effects such as material anisotropy, transverse shear and the primary and secondary cross-section warpings. The variable rotating speed and the arbitrary pitch angle influences were also considered in the presented formulation. The results demonstrated the important role of both the variable rotating speed and the arbitrary pitch angle consideration on the dynamics of the rotating beam. According to the knowledge of the author a few works are included the size effects for the rotating micro beams. Dehrouyeh-Semnani (Dehrouyeh-Semnani, 2015) developed the formulation for the in-plane free vibration of the rotating Timoshenko and Euler-Bernoulli micro beams. The flapping frequencies were obtained by the finite element method founded on the MCST. The results demonstrated the important effects of the material length scale, the rotation speed and the slenderness ratio on the flapping frequencies. Dehrouyeh-Semnani et al. (Dehrouyeh-Semnani et al. 2016) by considering the lead-lag motions, proposed the micro beam EOMs on the basis of the MCST and the assumptions of both the Timoshenko and the Euler-Bernoulli beam models. A similar analysis as their previous research paper, i.e. (Dehrouyeh-Semnani, 2015), was conducted to illustrate the important parameters in the determination of the natural frequencies.

The literature review demonstrate the lack of the researches on the micro beams by considering the size effects. Hence, in this paper, for the first time, a formulation for the rotating micro beams by employing the SGT and the Timoshenko and Euler-Bernoulli beam assumptions is developed. The outline of the paper is as follows. First by considering the Timoshenko and Euler-Bernoulli displacement fields the strain and kinetic energies are derived on the basis of the SGT. Using the Hamilton's principle the force and displacement forms of the governing equations and the accompanied BCs are obtained. By introducing the prestressed displacement components induced by the centrifugal forces, the EOMs and the corresponding BCs are linearized about the prestressed configuration. Some dimensionless parameters are addressed to nondimensionalize the EOMs and the associated BCs. The differential transform method (DTM) is applied on the EOMS and the corresponding BCs to extract the flapping and axial natural frequencies. After confirmation of the outcomes, some numerical studies are established to illustrate the material length scale parameter, the rotation speed, the shear deformation and the slenderness ratio influences on the flapping natural frequencies.

Section snippets

The equations of motion

To develop a formulation for a micro rotating Timoshenko beam, the displacement field components are assumed as (Arvin and Bakhtiari-Nejad, 2013):$u_x = u\left(x,t\right) + z\psi\left(x,t\right) \qquad u_z = w\left(x,t\right)$in which $u\left(x,t\right)$, $w\left(x,t\right)$ and $\psi\left(x,t\right)$ are, respectively, the axial, flapping and shear deformations and x and z axis are directed along the microbeam length and thickness, respectively. The strain energy for a linear elastic material which occupies a region with the volume $\mathscr{V}$, read as (Kong et al., 2009):$U = \frac{1}{2}\int_{\mathscr{V}}\Big(\sigma_{ij}\varepsilon_{ij} + m_{ij}^{s}\chi_{ij}^{s} + p_i\gamma$…

Linearization of the equations of motion

For linear vibration analysis, first the linearized EOMs are derived. As the rotating structures have a predeformed configuration due to the centrifugal forces, the EOMs are linearized about the mentioned configuration. To extract the predeformed configuration, the displacement components are assumed to have a static and a dynamic part (Arvin and Bakhtiari-Nejad, 2013). If the mircobeam is considered with a symmetric cross section made of an isotopic material, the only static displacement…

The dimensionless Timoshenko beam model equations of motion

After linearization of the EOMs, the following dimensionless parameters, $\widehat{w} = \frac{w}{L}$, $\widehat{\psi} = \psi$, $\widehat{u} = \frac{u}{L}$, $\widehat{x} = \frac{x}{L}$ and $\hat{t} = \frac{t}{T}$, are introduced to find the nondimensionalized EOMs and the BCs. After elimination of the ˆ sign and the subscript d, the dimensionless form of the EOMs and the associated BCs read as:$\partial_{xx} w u_s' + \partial_x w u_s'' + \kappa\left(\partial_{xx} w + \partial_x \psi\right)$ $\gamma_{\psi,\psi}\partial_{xx}\psi + \gamma_{\psi,w}\partial_{xxx} w - \eta^2\kappa\partial_x w + \left(\lambda_R^2 - \eta^2\kappa\right)\psi - \eta^2\beta_{10}$and

$- \alpha_{10}\left(u_s'''\partial_{xx} w + \partial_x w u_s^{(IV)}\right) + \gamma_{w,w}\partial_{xxxx} w + \gamma_{w,\psi}\partial_{xxx}\psi =$ $\partial_{xxxx}\psi = \partial_{tt}\psi,$

$\partial_{tt} w,$

$\partial_{xx} u - \alpha_{10}\partial_{xxxx} u = \partial_{tt} u - \lambda_R^2 u,and\partial_x w u_s' - \alpha_{10}\partial_x w u_s''' + \kappa\left(\partial_x w + \psi\right) + \gamma_{w,w}\partial_{xxx} w + \gamma_{w,\psi}\partial_{xx}\psi$…

Problem formulation by DTM

The differential transform method (see Lal and Ahlawat (2015)) is employed to acquire the flapping and axial natural frequencies. The most important standard rules of the DTM method for the EOMs and the BCs are introduced, respectively, in Tables A.1 and A.2 in Appendix A.…

Results validation

In this section, for validation of the formulation and the proposed solution some numerical analysis are presented. At first, a clamped-free stationary micro beam is considered with the material and geometric specifications as $E = 1.44GPa$, $\rho = 1000kg/m^3$, $\bar{\nu} = 0.38$ and the material length scales, $l_0 = l_1 = l_2 = 17.6\mu m$. The micro beam length and width ratio with respect to the thickness are, respectively, $L/h = 20$ and $b/h = 2$ (Kong et al., 2009). Comparison of the first three flapping natural frequencies are…

Conclusions

In this paper, the EOMs for a rotating micro beam on the basis of strain gradient theory and the assumptions of both Timoshenko and Euler-Bernoulli beam models were developed. By separation of the axial displacement to a static part which is induced by the centrifugal forces and a dynamic part that models the vibration about the static configuration, for a symmetric isotropic micro beam, the EOMs were linearized. Some dimensionless parameters were introduced for nondimensionalization of the…

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> …Recently, Arvin developed their previous studies for considering the length scale effects in rotating microscale beams. He derived the governing equations of rotating microbeams on the basis of the modified couple stress theory [27] and the strain gradient theory [28]. He studied the effects of the material length scale parameter as well as the slenderness ratio on the natural frequencies.…

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