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A novel fuzzy logic Levenberg-Marquardt method to solve the ill-conditioned power flow problem

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ABSTRACT

This paper presents a new three-step Levenberg-Marquardt (TSLM) algorithm by using fuzzy logic theory (FLT) for solving power flow equations in ill-conditioned power systems. Using the proposed fuzzy TSLM (FTSLM) method reduces computation times and the number of iterations. In most cases, the FTSLM method converges in the first iteration due to its biquadratic convergence. The reactive and real power mismatches at each bus of the electrical power system are chosen as the input values for fuzzification. The output values of the FTSLM (after defuzzification) are voltage magnitudes and angles of the buses. The proposed method is tested on standard ill-conditioned 11-bus, 13-bus, 43-bus, 118-bus and 2383-bus test systems and the results are compared with the benchmark methods.

1. Introduction

The equations of the AC power flow problem are modeled by a set of non-linear algebraic equations. Conventionally, the power flow problem is solved using Newton or Newton-based techniques to determine the voltage angles, voltage magnitudes and line flows of a power system. Solving the power flow problem is a fundamental requirement for the analysis of the power systems and is used in real time operation and to control systems [1,2].

Since the presentation of digital computers, many attempts have been made to solve the power flow problem using several techniques. Ref. [3] has worked on the convergence characteristics of different techniques. Some famous methods in this area are Gauss-Seidel method, Newton method and decoupled and fast-decoupled methods all of which are used for well-conditioned power system. These types of systems are the most common cases in the power systems and the solution of the power flow equations exists [4–7].

Other types of power systems are ill-conditioned systems, in which the solution of the power flow equations exists, but using traditional methods may fail to converge or have slow convergence rates. Bad or unfit selection of the swing bus, very high $\frac{R_{line}}{X_{line}}$ ratio of lines and heavy loading of power systems are common reasons why this situation occurs [8,9]. In such cases, the failure of traditional techniques is due to the instability of the techniques, but not due to the instability of the nonlinear power flow equations [10,11].

Ref. [12] has solved the optimal power flow problem in an ill-conditioned power system using quadratic discriminant index. This method can find a low voltage solution at the maximum loading point. Authors in [13] have applied a corrected Levenberg-Marquardt algorithm with a non-monotone line search to solve the power flow problem in the ill-conditioned systems. An asynchronous parallel computing has been used to solve the power flow problem of the ill-conditioned rural distribution systems in [14]. Ref. [15] has presented a robust method to solve the power flow problem in the ill-conditioned power system using a high-order predictor of the asymptotic numerical method and homotopy transformation.

In 1965, Zadeh has presented the foundations of the fuzzy set theory [16] as a technique for dealing with the imprecision of practical systems. Many literatures have worked on the topic of the fuzzy logic and application of this logic for solving problems in power systems since Zadeh recommended the fuzzy set and concept of fuzzy logic [17].

The fuzzy logic theory (FLT) has been implemented in many power system processes as a foundation. This implementation is due to some reasons such as:

- Improving robustness in comparison with traditional techniques
- Increasing computing speed

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- The simplification of power system modeling

FLT is used for solving a wide area of problems in power systems such as power system control and operation, load forecasting, power system planning and power system stability [18–20]. In addition, this technique has been used in various types to solve the power flow problem [23–26]. The loads of power systems are variable and uncertain. Using the FLT can handle this uncertainty. If the input data of the power flow problem such as generation power (active and reactive powers) and the loads are given as fuzzy sets, the problem can be solved using FLT [21]. Other uncertain input data in the power flow calculation can be handled in the same way. Using FLT to consider the uncertainty in power systems were presented in some literatures such as [22–24]. Ref. [25] has presented a framework for solving AC and DC power flows and considered the uncertainty in power generations and loads. In [26], a fuzzy based approach has been presented for the adjustment of variable parameters in power flow studies. These variables are phase angles, transformers tap positions and line impedances. In [27], input values for the fuzzy logic are reactive power ($Q_{reactive}$) and real power (P_{active}) mismatches per voltage magnitudes ($|V|$) at each bus. These values are fuzzified in the fuzzifier and the output values are the correction of the voltage magnitudes ($|V|$) and voltage angles (δ) at each bus. In [28], a fuzzy-based approach has been presented and used to solve the power flow problem. Authors of [28] have used Gaussian membership functions for fuzzification that can reduce the number of iterations in comparison with the triangular membership functions. Authors in [29,30] have presented the symmetric fuzzy power flow problem and solved it as an optimization problem constrained to the power flow equations. In [31], authors have presented a fuzzy technique to solve the power flow in unbalanced and balanced radial distribution systems. This technique needs the bus injection to branch current (BIBC) and branch current to bus voltage (BCBV) matrices of the network and uses the voltage of the substation and the power demands as fuzzy inputs. Ref. [32] has solved the power flow problem using combination of fuzzy simulation and connection number while considering the uncertainties of loads and renewable generations.

The main contribution of this paper is to present a novel fuzzy-based three-step Levenberg-Marquardt (FTSLM) algorithm for solving the power flow problem in the ill-conditioned power systems. Application of the proposed method can significantly reduce the computation time and also the number of iterations in solving the power flow problems in the ill-conditioned power systems.

The presented approach can be used in small to large power systems. The following sections are designed to introduce the presented approach. In ection 2, the concept of the ill-conditioned power systems is reviewed. Section 3 presents the mathematical formulation of the TSLM method and ection 4 applies this method to solve the power flow problem. Section 5 presents the combination of TSLM algorithm and FLT to solve the ill-conditioned systems. Section 6 presents the case studies and numerical results. Section 7 discusses the relations between voltage instability, loading condition of the system and occurrence of the ill-condition state in a power system. Finally, the conclusions are presented in ection 8.

2. Introduction of the ill-conditioned power systems

The main aim of solving the power flow problem is to find the voltage magnitude and angle at the buses of the power system. Eqs. (1) and (2) show the relation between the injected powers and the magnitude and angle of voltage at each bus.

$$P_k = |V_k| \sum_{j=1}^{N} |V_j| (G_{kj} \cos\theta_{kj} + B_{kj} \sin\theta_{kj}) \tag{1}$$

$$Q_k = |V_k| \sum_{j=1}^{N} |V_j| (G_{kj} \sin\theta_{kj} - B_{kj} \cos\theta_{kj}) \tag{2}$$

where

$$V_k = |V_k| (\cos\theta_k + j\sin\theta_k) \tag{3}$$

The above equations are nonlinear and can be solved by the Taylor series and using first-order series approximation [33]:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \left(\frac{\partial P}{\partial \theta}\right) & \left(\frac{\partial P}{\partial |V|}\right) \\ \left(\frac{\partial Q}{\partial \theta}\right) & \left(\frac{\partial Q}{\partial |V|}\right) \end{bmatrix} * \begin{bmatrix} \Delta\theta \\ \Delta|V| \end{bmatrix} \tag{4}$$

In Eq. (4), we take the Jacobian matrix as:

$$[\mathcal{J}] = \begin{bmatrix} \left(\frac{\partial P}{\partial \theta}\right) & \left(\frac{\partial P}{\partial |V|}\right) \\ \left(\frac{\partial Q}{\partial \theta}\right) & \left(\frac{\partial Q}{\partial |V|}\right) \end{bmatrix} \tag{5}$$

Von Neumann has proposed an index for classification of the condition of a system [34]. This index is condition number (*CN*) and defined by Eq. (6).

$$CN = \frac{\pi_{\max}}{\pi_{\min}} \tag{6}$$

where π_{max} and π_{min} are the largest (maximum) and smallest (minimum) eigenvalues of the Jacobian matrix ($[\mathcal{J}]$), respectively. A large value of *CN* illustrates the ill-condition situation of the system [34–37]. Ref. [9] has studied this index in power systems and stated that threshold value for starting the ill-condition state of a system is $CN = 0.1086*10^4$.

3. Three-Step Levenberg-Marquardt (TSLM) algorithm

This section presents the mathematical base of the proposed method. In the ill-conditioned power systems using the traditional methods such as Newton-like methods may cause to diverge. This situation is due to sensitivity of the solution the round-off errors injected during computation [38].

Consider a system with the algebraic nonlinear equations, as follows:

$$F_i(x_1, x_2, \dots, x_n) = 0 \quad i = 1, 2, \dots, n \tag{7}$$

When the Jacobian matrix of Eq. (7) is near singular, the system is named "ill-conditioned" and traditional methods such as Newton method are not suitable for solving the equations. In such situations, applying the TSLM algorithm can be useful.

The TSLM algorithm calculates Eq. (8) at each iteration [38]:

$$\varphi_{(k)}^{LM} = -\left(\mathcal{J}_{(k)}^T \mathcal{J}_{(k)} + \gamma_{(k)} I\right)^{-1} \mathcal{J}_{(k)}^T \mathcal{F}_{(k)} \tag{8}$$

where $\mathcal{F}_{(k)} = \mathcal{F}(x_{(k)})$, $\mathcal{J}_{(k)}$ is Jacobian matrix and $\gamma_{(k)}$ is the "convergence parameter" that is calculated by:

$$\gamma_{(k)} = \|\mathcal{F}(x_{(k)})\|^2 \tag{9}$$

To apply the TSLM algorithm, the following steps will be taken [38]:

Set the initial guess and $k = 1$

Step 1:

If: $\left\| \mathscr{J}_{(k)}^{T} \mathscr{F}_{(k)} \right\| = 0$ then stop; otherwise calculate $\varphi_{(k)}^{LM}$, $\varphi_{(k)}^{MLM}$ and $\varphi_{(k)}^{HLM}$ by Eqs. (10)–(14).

Step 2:

Set:

$$\varphi_{(k)}^{LM} = -\left(\mathscr{J}_{(k)}^{T}\mathscr{J}_{(k)} + \gamma_{(k)} I\right)^{-1} \mathscr{J}_{(k)}^{T} \mathscr{F}(x_k) \tag{10}$$

$$y_k = x_k + \varphi_{(k)}^{LM} \tag{11}$$

$$\varphi_{(k)}^{MLM} = -\left(\mathscr{J}_{(k)}^{T}\mathscr{J}_{(k)} + \gamma_{(k)} I\right)^{-1} \mathscr{J}_{(k)}^{T} \mathscr{F}(y_k) \tag{12}$$

$$z_k = y_k + \varphi_{(k)}^{MLM} \tag{13}$$

$$\varphi_{(k)}^{HLM} = -\left(\mathscr{J}_{(k)}^{T}\mathscr{J}_{(k)} + \gamma_{(k)} I\right)^{-1} \mathscr{J}_{(k)}^{T} \mathscr{F}(z_k) \tag{14}$$

Step 3:

Calculate w_k using Eq. (15):

$$w_k = \varphi_{(k)}^{LM} + \eta_k \varphi_{(k)}^{MLM} + \varphi_{(k)}^{HLM} \tag{15}$$

In Eq. (15) η_k is step size of proposed method [38].

Set:

$$x_{k+1} = x_k + w_k \tag{16}$$

If $\max(abs(\mathscr{F}(y_k))) < \varepsilon$ then stop; Otherwise

Refresh the converge parameter ($\gamma_{(k)}$)
Set $k = k + 1$ and go back to Step 1.

4. Application of the TSLM algorithm to solve the power flow in the Ill-conditioned systems

In this section, a proper algorithm for solving the power flow equations in an ill-conditioned power system has been presented. The presented algorithm is based on the TSLM method, which was introduced in the previous section. To facilitate programming, the following steps are designed:

Step 1:

Input the necessary data of the power system (such as lines' and buses' data) and set the initial guesses and $k = 1$.

Step 2:

Calculate $\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}_k$ using $\begin{bmatrix}\delta\\ |V|\end{bmatrix}_k$ and calculate $\begin{bmatrix}\delta\\ |V|\end{bmatrix}_{(LM)-k}$ by Eqs. (17) and (18).

$$\begin{bmatrix}\Delta\delta\\ \Delta|V|\end{bmatrix}_{(LM)-k} = -\left(\mathscr{J}_{(k)}^{T}\mathscr{J}_{(k)} + \gamma_{(k)} I\right)^{-1} \mathscr{J}_{(k)}^{T} \begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}_k \tag{17}$$

$$\begin{bmatrix}\delta\\ |V|\end{bmatrix}_{(LM)-k} = \begin{bmatrix}\Delta\delta\\ \Delta|V|\end{bmatrix}_{(LM)-k} + \begin{bmatrix}\delta\\ |V|\end{bmatrix}_k \tag{18}$$

Calculate $\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}_{(LM)-k}$ using $\begin{bmatrix}\delta\\ |V|\end{bmatrix}_{(LM)-k}$ and calculate $\begin{bmatrix}\delta\\ |V|\end{bmatrix}_{(MLM)-k}$ by Eqs. (19) and (20)

$$\begin{bmatrix}\Delta\delta\\ \Delta|V|\end{bmatrix}_{(MLM)-k} = -\left(\mathscr{J}_{(k)}^{T}\mathscr{J}_{(k)} + \gamma_{(k)} I\right)^{-1} \mathscr{J}_{(k)}^{T} \begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}_{(LM)-k} \tag{19}$$

$$\begin{bmatrix}\delta\\ |V|\end{bmatrix}_{(MLM)-k} = \begin{bmatrix}\Delta\delta\\ \Delta|V|\end{bmatrix}_{(MLM)-k} + \begin{bmatrix}\delta\\ |V|\end{bmatrix}_{(LM)-k} \tag{20}$$

Calculate $\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}_{(MLM)-k}$ using $\begin{bmatrix}\delta\\ |V|\end{bmatrix}_{(MLM)-k}$ and calculate $\begin{bmatrix}\Delta\delta\\ \Delta|V|\end{bmatrix}_{(HLM)-k}$ by Eq. (21)

$$\begin{bmatrix}\Delta\delta\\ \Delta|V|\end{bmatrix}_{(HLM)-k} = -\left(\mathscr{J}_{(k)}^{T}\mathscr{J}_{(k)} + \gamma_{(k)} I\right)^{-1} \mathscr{J}_{(k)}^{T} \begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}_{(MLM)-k} \tag{21}$$

Step 3:

Calculate $\begin{bmatrix}\Delta\delta\\ \Delta|V|\end{bmatrix}_{k+1}$ by Eq. (22) and calculate $\begin{bmatrix}\delta\\ |V|\end{bmatrix}_{k+1}$ by Eq. (23):

$$\begin{bmatrix}\Delta\delta\\ \Delta|V|\end{bmatrix}_{k+1} = \begin{bmatrix}\Delta\delta\\ \Delta|V|\end{bmatrix}_{(LM)-k} + \eta_k \begin{bmatrix}\Delta\delta\\ \Delta|V|\end{bmatrix}_{(MLM)-k} + \begin{bmatrix}\Delta\delta\\ \Delta|V|\end{bmatrix}_{(HLM)-k} \tag{22}$$

$$\begin{bmatrix}\delta\\ |V|\end{bmatrix}_{k+1} = \begin{bmatrix}\delta\\ |V|\end{bmatrix}_{(k)} + \begin{bmatrix}\Delta\delta\\ \Delta|V|\end{bmatrix}_{k+1} \tag{23}$$

If $\max\left(abs\left(\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}_{(MLM)-k}\right)\right) < \varepsilon$ then stop; otherwise:

Set $k = k + 1$ and go back to Step 2.

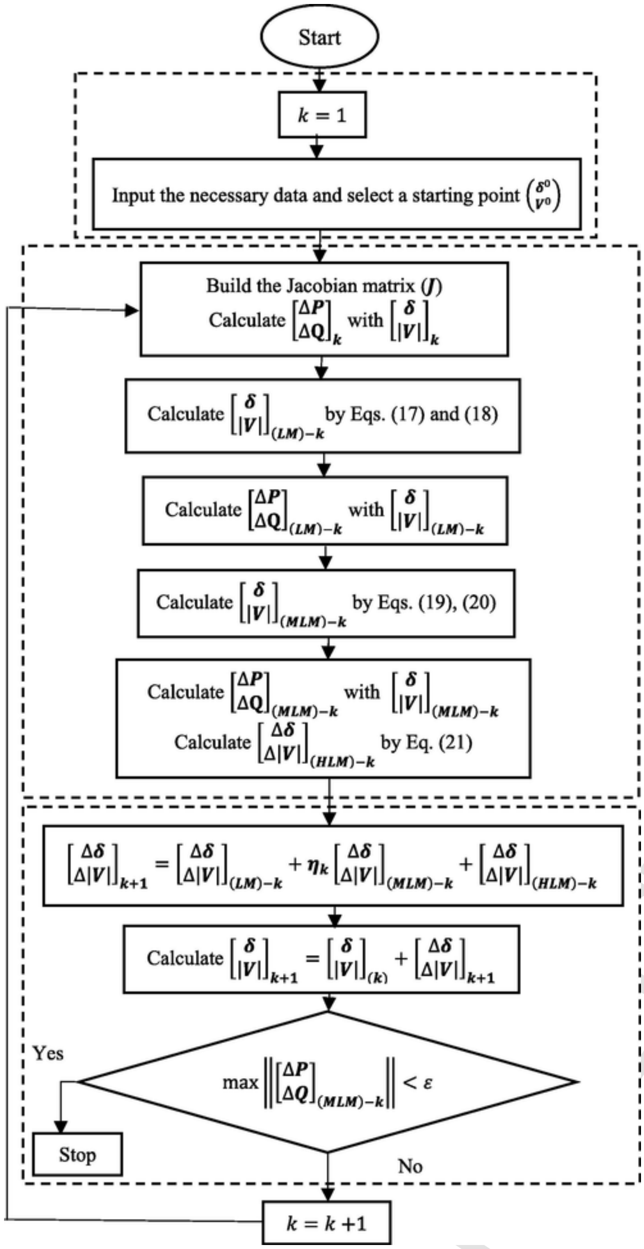

Fig. 1. Flowchart of the TSLM method.

Fig. 1 shows flowchart of the TSLM method for solving the power flow problem of the ill-conditioned power systems.

5. Combination of the TSLM algorithm and fuzzy logic to solve the Ill-conditioned systems

This section combines the TLSM and fuzzy logic to present a method that is named fuzzy TSLM (FTSLM). The presented method is suitable to solve the power flow equations of the ill-conditioned power systems.

The FTSLM is based on the TSLM introduced in the previous section, but the continuous update of the state vector of the power system ($\begin{bmatrix}\delta\\ |V|\end{bmatrix}$) is calculated via the fuzzy logic concept instead of using traditional power flow methods.

Eq. (24) is used for implementation of the FLT in solving the power flow problem in the ill-conditioned power systems:

$$\begin{bmatrix}\Delta\delta\\ \Delta|V|\end{bmatrix} = fuz\left(\begin{bmatrix}\Delta P\\ \Delta \mathrm{Q}\end{bmatrix}\right) \tag{24}$$

In the proposed FTSLM method, inputs consist of $\Delta S_k^{(1)}$, $\Delta S_k^{(2)}$ and $\Delta S_k^{(3)}$ (ΔS_k^i consists of reactive and real power mismatches) and outputs are $\varphi_{(k)}^{LM}$, $\varphi_{(k)}^{MLM}$ and $\varphi_{(k)}^{HLM}$. Note that $\varphi_{(k)}^{LM}$, $\varphi_{(k)}^{MLM}$ and $\varphi_{(k)}^{HLM}$ consist of voltage angles' and voltage magnitudes' mismatches.

$$\begin{cases}\varphi_{(k)}^{LM} = fuz(\Delta S_k^{(1)})\\ \varphi_{(k)}^{MLM} = fuz(\Delta S_k^{(2)})\\ \varphi_{(k)}^{HLM} = fuz(\Delta S_k^{(3)})\end{cases} \tag{25}$$

The main structure of the proposed FTSLM is shown in Fig. 2. The design process of the fuzzy inference system (FIS) has five steps:

(1) The fuzzification of the variables of the TSLM
(2) The definition of the membership function
(3) The creation of rule base of fuzzy model
(4) The process of fuzzy logic
(5) The defuzzification of variables of the TSLM

5.1. First inputs -first outputs

The first input signals ($\Delta S_k^{(1)}$) are fuzzified into active and reactive fuzzy signals ($\Delta S_{k-P}^{(1)}$ and $\Delta S_{k-Q}^{(1)}$) with 7 variables. Also, the first output signals ($\varphi_{(k)}^{LM}$) are fuzzified into phase and amplitude of voltage fuzzy signals ($\varphi_{(k-\delta)}^{LM}$ and $\varphi_{(k-|V|)}^{LM}$) represented in triangular function form consists of large negative (LN), medium negative (MN), small negative (SN), zero (ZR), small positive (SP), medium positive (MP), large positive (LP). Table 1 and Figs. 3 and 4 show the membership function of the first input and output.

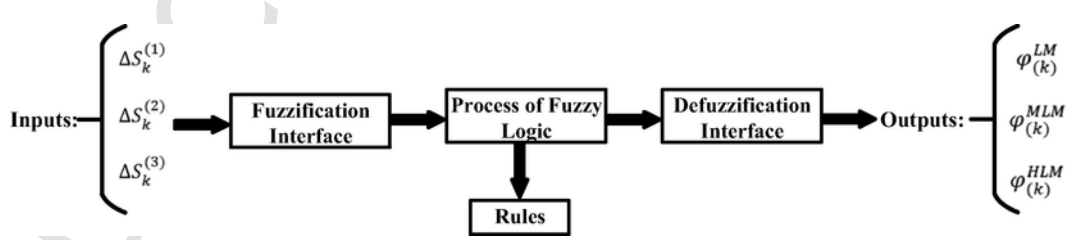

Fig. 2. Fuzzy inference system of the FTSLM.

Table 1
First membership functions.

First input signals $\mu(\Delta S_k^{(1)})$	First output signals $\mu(\varphi_{(k)}^{LM})$
$LN : \left[-\infty.-\Delta S_{k_{max}}^{(1)}.\frac{-\Delta S_{k_{max}}^{(1)}}{3.75}\right]$	$LN : \left[-\infty.-\varphi_k^{LM}.\frac{-\varphi_k^{LM}}{3.75}\right]$
$MN : \left[-\Delta S_{k_{max}}^{(1)}.\frac{-\Delta S_{k_{max}}^{(1)}}{2}.0\right]$	$MN : \left[-\varphi_k^{LM}.\frac{-\varphi_k^{LM}}{2}.0\right]$
$SN : \left[\frac{-\Delta S_{k_{max}}^{(1)}}{3.75}.\frac{-\Delta S_{k_{max}}^{(1)}}{7.5}.0\right]$	$SN : \left[\frac{-\varphi_k^{LM}}{3.75}.\frac{-\varphi_k^{LM}}{7.5}.0\right]$
$ZR : \left[\frac{-\Delta S_{k_{max}}^{(1)}}{15}.0.\frac{\Delta S_{k_{max}}^{(1)}}{15}\right]$	$ZR : \left[\frac{-\varphi_k^{LM}}{15}.0.\frac{\varphi_k^{LM}}{15}\right]$
$SP : \left[0.\frac{\Delta S_{k_{max}}^{(1)}}{7.5}.\frac{\Delta S_{k_{max}}^{(1)}}{3.75}\right]$	$SP : \left[0.\frac{\varphi_k^{LM}}{7.5}.\frac{\varphi_k^{LM}}{3.75}\right]$
$MP : \left[0.\frac{\Delta S_{k_{max}}^{(1)}}{2}.\Delta S_{k_{max}}^{(1)}\right]$	$MP : \left[0.\frac{\varphi_k^{LM}}{2.5}.\varphi_k^{LM}\right]$
$LP : \left[\frac{\Delta S_{k_{max}}^{(1)}}{3.75}.\Delta S_{k_{max}}^{(1)}.+\infty\right]$	$LP : \left[\frac{\varphi_k^{LM}}{3.75}.\varphi_k^{LM}.+\infty\right]$

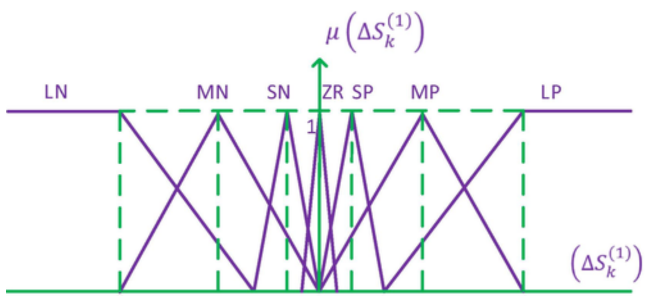

Fig. 3. Membership function of the first input signals ($\Delta S_k^{(1)}$).

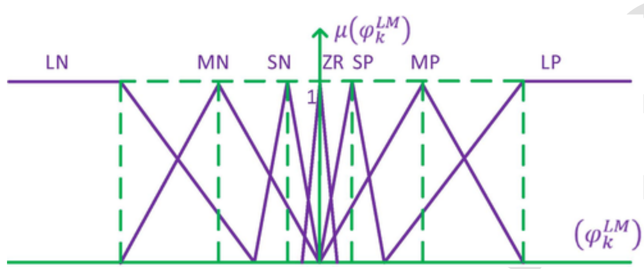

Fig. 4. Membership function of the first output signals ($\varphi_{(k)}^{LM}$).

5.2. Second inputs – second outputs

The second input signals ($\Delta S_k^{(2)}$) are fuzzified into active and reactive fuzzy signals ($\Delta S_{k-P}^{(2)}$ and $\Delta S_{k-Q}^{(2)}$) with 5 variables and the first output signals ($\varphi_{(k)}^{MLM}$) are fuzzified into phase and amplitude of voltage fuzzy signals ($\varphi_{(k-\delta)}^{MLM}$ and $\varphi_{(k-|V|)}^{MLM}$) represented in triangular function form consists of large negative (LN), small negative (SN), zero (ZR), small positive (SP), large positive (LP). Table 2 and Figs. 5 and 6 show the membership function of the second input and output.

5.3. Third inputs – third outputs

As shown in Table 3 and Figs. 7 and 8, the third input ($\Delta S_k^{(3)}$) signals are fuzzified into active and reactive fuzzy signals ($\Delta S_{k-P}^{(3)}$ and $\Delta S_{k-Q}^{(3)}$) with 3 variables. The similar method has been applied for the third output signals ($\varphi_{(k)}^{HLM}$).

Table 2
Second membership functions.

Second input signals $\mu(\Delta S_k^{(2)})$	Second output signals $\mu(\varphi_{(k)}^{MLM})$
$LN : \left[-\infty.-\Delta S_{k_{max}}^{(2)}.\frac{-\Delta S_{k_{max}}^{(2)}}{3.75}\right]$	$LN : \left[-\infty.-\varphi_k^{MLM}.\frac{-\varphi_k^{MLM}}{3.75}\right]$
$SN : \left[-\Delta S_{k_{max}}^{(2)}.\frac{-\Delta S_{k_{max}}^{(2)}}{2}.0\right]$	$SN : \left[-\varphi_k^{MLM}.\frac{-\varphi_k^{MLM}}{2}.0\right]$
$ZR : \left[\frac{-\Delta S_{k_{max}}^{(2)}}{3.75}.0.\frac{\Delta S_{k_{max}}^{(2)}}{3.75}\right]$	$ZR : \left[\frac{-\varphi_k^{MLM}}{3.75}.0.\frac{\varphi_k^{MLM}}{3.75}\right]$
$SP : \left[0.\frac{\Delta S_{k_{max}}^{(2)}}{2}.-\Delta S_{k_{max}}^{(2)}\right]$	$SP : \left[0.\frac{\varphi_k^{MLM}}{2}.\varphi_k^{MLM}\right]$
$LP : \left[\frac{\Delta S_{k_{max}}^{(2)}}{3.75}.\Delta S_{k_{max}}^{(2)}.+\infty\right]$	$LP : \left[\frac{\varphi_k^{MLM}}{3.75}.\varphi_k^{MLM}.+\infty\right]$

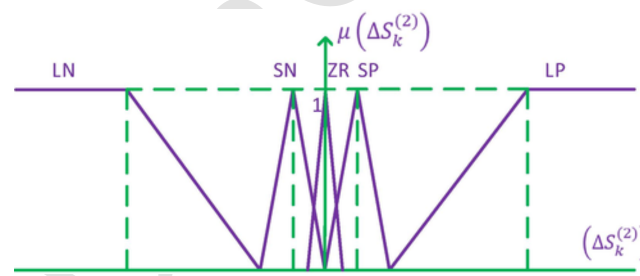

Fig. 5. Membership function of the second input signals ($\Delta S_k^{(2)}$).

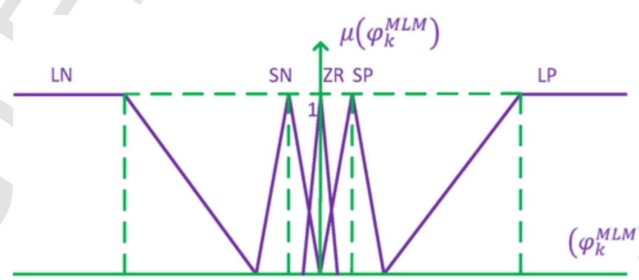

Fig. 6. Membership function of the first output signals ($\varphi_{(k)}^{MLM}$).

Table 3
Third membership functions.

Third input signals $\mu(\Delta S_k^{(3)})$	Third output signals $\mu(\varphi_{(k)}^{HLM})$
$LN : \left[-\infty.-\Delta S_{k_{max}}^{(3)}.\frac{-\Delta S_{k_{max}}^{(3)}}{5}\right]$	$LN : \left[-\infty.-\varphi_k^{HLM}.\frac{-\varphi_k^{HLM}}{5}\right]$
$ZR : \left[\frac{-\Delta S_{k_{max}}^{(3)}}{1.5}.0.\frac{\Delta S_{k_{max}}^{(3)}}{1.5}\right]$	$ZR : \left[\frac{-\varphi_k^{HLM}}{1.5}.0.\frac{\varphi_k^{HLM}}{1.5}\right]$
$LP : \left[\frac{\Delta S_{k_{max}}^{(3)}}{5}.\Delta S_{k_{max}}^{(3)}.+\infty\right]$	$LP : \left[\frac{\varphi_k^{HLM}}{5}.\varphi_k^{HLM}.+\infty\right]$

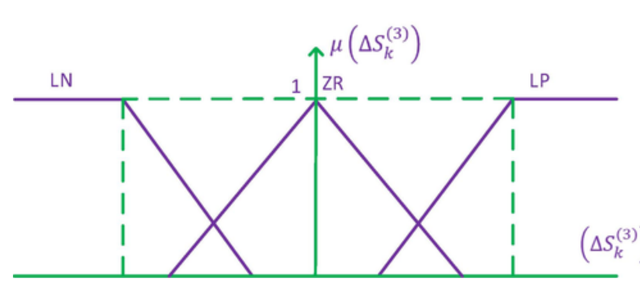

Fig. 7. Membership function of the third input signals ($\Delta S_k^{(3)}$).

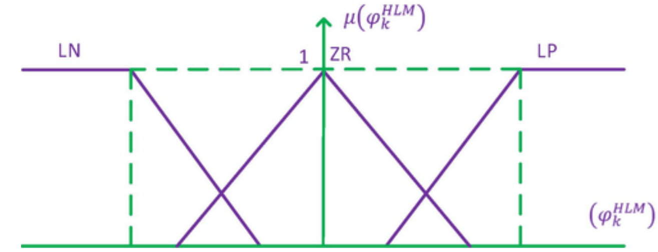

Fig. 8. Membership function of the third output signals ($\varphi_{(k)}^{HLM}$).

For the proposed FTSLM method, the rules are listed as follows:

$$
\begin{aligned}
&\boldsymbol{R}1:\ \textit{IF}\ \Delta S_k^{(1)}\ \textit{is LN then}\ \varphi_k^{LM}\ \textit{is LN}\\
&\boldsymbol{R}2:\ \textit{IF}\ \Delta S_k^{(1)}\ \textit{is MN then}\ \varphi_k^{LM}\ \textit{is MN}\\
&\boldsymbol{R}3:\ \textit{IF}\ \Delta S_k^{(1)}\textit{is SN then}\ \varphi_k^{LM}\ \textit{is SN}\\
&\boldsymbol{R}4:\ \textit{IF}\ \Delta S_k^{(1)}\textit{is ZR then}\ \varphi_k^{LM}\ \textit{is ZR}\\
&\boldsymbol{R}5:\ \textit{IF}\ \Delta S_k^{(1)}\textit{is SP then}\ \varphi_k^{LM}\ \textit{is SP}\\
&\boldsymbol{R}6:\ \textit{IF}\ \Delta S_k^{(1)}\ \textit{is MP then}\ \varphi_k^{LM}\ \textit{is MP}\\
&\boldsymbol{R}7:\ \textit{IF}\ \Delta S_k^{(1)}\ \textit{is LP then}\ \varphi_k^{LM}\ \textit{is LP}\\
&\boldsymbol{R}8:\ \textit{IF}\ \Delta S_k^{(2)}\ \textit{is LN then}\ \varphi_k^{MLM}\textit{is LN}\\
&\boldsymbol{R}9:\ \textit{IF}\ \Delta S_k^{(2)}\ \textit{is SN then}\ \varphi_k^{MLM}\ \textit{is SN}\\
&\boldsymbol{R}10:\ \textit{IF}\ \Delta S_k^{(2)}\ \textit{is ZR then}\ \varphi_k^{MLM}\ \textit{is ZR}\\
&\boldsymbol{R}11:\ \textit{IF}\ \Delta S_k^{(2)}\ \textit{is SP then}\ \varphi_k^{MLM}\ \textit{is SP}\\
&\boldsymbol{R}12:\ \textit{IF}\ \Delta S_k^{(2)}\ \textit{is LP then}\ \varphi_k^{MLM}\ \textit{is LP}\\
&\boldsymbol{R}13:\ \textit{IF}\ \Delta S_k^{(3)}\ \textit{is LN then}\ \varphi_k^{HLM}\ \textit{is LN}\\
&\boldsymbol{R}14:\ \textit{IF}\ \Delta S_k^{(3)}\ \textit{is ZR then}\ \varphi_k^{HLM}\ \textit{is ZR}\\
&\boldsymbol{R}15:\ \textit{IF}\ \Delta S_k^{(3)}\ \textit{is LP then}\ \varphi_k^{HLM}\ \textit{is LP}\\
&\boldsymbol{R}16:\ \textit{IF}\ \Delta S_k^{(1)}\ \textit{and}\ \Delta S_k^{(2)}\textit{are SN and}\ \Delta S_k^{(3)}\ \textit{is LN then}\ \varphi_k^{HLM}\\
&\boldsymbol{R}17:\ \textit{IF}\ \Delta S_k^{(1)}\ \textit{and}\ \Delta S_k^{(2)}\ \textit{are ZR and}\ \Delta S_k^{(3)}\ \textit{is LN then}\ \varphi_k^{HLM}\\
&\boldsymbol{R}18:\ \textit{IF}\ \Delta S_k^{(1)}\ \textit{and}\ \Delta S_k^{(2)}\ \textit{are SP and}\ \Delta S_k^{(3)}\ \textit{is LN then}\ \varphi_k^{HLM}
\end{aligned}
$$

The number of triangular fuzzy-membership functions applied in proposed method and fuzzy rules are selected based on [27,28].

5.4. Consideration of different types of loads in FTSLM method

The power systems have several types of loads including constant impedances (Z), constant current (I) and constant power (P) loads. These types of loads (ZIP loads) are modeled, as follows [35]:

$$P_{Di}(V_i) = P_{0i}\left[\alpha_{0i}\left(\frac{V_i}{V_{0i}}\right)^2 + \alpha_{1i}\left(\frac{V_i}{V_{0i}}\right) + \alpha_{2i}\right] \tag{26}$$

$$Q_{Di}(V_i) = Q_{0i}\left[\beta_{0i}\left(\frac{V_i}{V_{0i}}\right)^2 + \beta_{1i}\left(\frac{V_i}{V_{0i}}\right) + \beta_{2i}\right] \tag{27}$$

where parameters (α_{0i},β_{0i}), (α_{1i},β_{1i}) and (α_{2i},β_{2i}) model the constant impedance, constant current and constant power loads, respectively. So, based on [35], to calculate the power residuals in the k-th iteration ($\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}_k, \begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}_{(LM)-k}, \begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}_{(MLM)-k}$ in Eqs. (17), (19) and (21)), scheduled active and reactive powers should be considered as follows:

$$
\begin{aligned}
P_i^{sch,(k)}(V_i^{(k)}) &= P_{G_i} - P_{Di}(V_i^{(k)})\\
&= P_{G_i} - P_{0i}\left[\alpha_{0i}\left(\frac{V_i^{(k)}}{V_{0i}}\right)^2 + \alpha_{1i}\left(\frac{V_i^{(k)}}{V_{0i}}\right) + \alpha_{2i}\right]
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
Q_i^{sch,(k)}(V_i^{(k)}) &= Q_{G_i} - Q_{Di}(V_i^{(k)})\\
&= Q_{G_i} - Q_{0i}\left[\beta_{0i}\left(\frac{V_i^{(k)}}{V_{0i}}\right)^2 + \beta_{1i}\left(\frac{V_i^{(k)}}{V_{0i}}\right) + \beta_{2i}\right]
\end{aligned} \tag{29}
$$

6. Case studies and numerical results

To evaluate the effectiveness of the proposed FTSLM method, five ill-conditioned power systems have been selected. These test systems are 11-bus, 13-bus (a distribution test system), 43-bus, 118-bus and 2383-bus power systems [9,39,40].

Table 4 shows π_{min} and π_{max} of the Jacobian matrix (Eq. (6)) and also *CN* of the studied power systems. As shown in this table, the condition numbers of the test systems are sufficiently high and these systems are ill-conditioned.

The proposed FTSLM method has been implemented using MATLAB/Simulink software by a PC with 2.5 GHz CPU and 4 GB RAM. The presented CPU times are the average CPU times of the calculation procedures (Each case study has been run 200 times and the average time has been obtained).

Table 5 shows the results of solving the power flow equations in the ill-conditioned 11-bus power system. The input columns in Table 4 show the values of $\Delta S_k^{(1)}$, $\Delta S_k^{(2)}$ and $\Delta S_k^{(3)}$ at each bus and the output columns represent the values of $\varphi_{(k)}^{LM}$, $\varphi_{(k)}^{MLM}$ and $\varphi_{(k)}^{HLM}$. The last columns show the voltage magnitudes and phases of the buses. Similar studies have been done on the other test systems, but for the sake of brevity, the similar simulations results are not presented in this form.

In order to evaluate the effectiveness of the proposed method, the FTSLM and TSLM methods have been compared with three famous algorithms, which are used to solve the power flow equations in the ill-conditioned power systems. These benchmark algorithms are Iterative Regularization Newton (IRN) method [41], LM method [42] and Newton Raphson–Jacobian Marquardt (NRJM) method [43,44]. The number of iterations and the average execution time of solving the power flow equations of the 11-bus, 13-bus, 43-bus, 118-bus and 2383-bus

Table 4
Maximum and minimum eigenvalues and the condition numbers of the case studies.

Case study	Min. Eigenvalue ($\pi_{\min}$)	Max. Eigenvalue ($\pi_{\max}$)	*CN*
11-Bus	0.1126	$0.1222*10^3$	$0.1086*10^4$
13-Bus	$0.1442*10^{-1}$	$0.2905*10^2$	$0.2014*10^4$
43-Bus	$0.9476*10^{-1}$	$0.2426*10^4$	$0.2560*10^5$
118-Bus	$0.726*10^{-1}$	$0.575*10^3$	$0.7922*10^4$
2383-Bus	0.0003	$6.9067*10^8$	$2.3022*10^{12}$

Table 5
Results of implementation of the proposed method on the 11-bus test system.

BUS	Inputs			Outputs			$\|V\|$	δ (rad)
	$(\Delta S_k^{(1)})$	$(\Delta S_k^{(2)})$	$(\Delta S_k^{(3)})$	$(\varphi_{(k)}^{LM})$	$(\varphi_{(k)}^{MLM})$	$(\varphi_{(k)}^{HLM})$		
1	–	–	–	–	–	–	1.024	0
2	0.3399	−0.0072	0.0002	0.0219	−0.0002	0.0001	1.022	−0.221
3	−0.0062	0.001	0	0.0201	0.0001	0.0001	1.0204	−0.404
4	0	0.0007	0	0.02	−0.0001	0	1.02	−0.265
5	−0.008	0.0011	0	0.0182	0.0002	0	1.0185	−0.460
6	−0.0068	0.0009	0	0.0199	−0.0001	0	1.0199	−0.274
7	0	0.0012	0.0006	0.0059	−0.0003	−0.0004	1.003	−0.873
8	0	0.0012	0.0007	0.0022	−0.0004	−0.0005	0.9985	−1.039
9	−0.0009	0.0012	0.0008	0.0006	−0.0001	−0.0004	0.9977	−1.143
10	0	0.0009	0.0009	−0.0061	−0.0012	−0.0007	0.9887	−1.416
11	−0.0057	0.0007	0.0009	−0.0082	−0.0013	−0.0007	0.9867	−1.652

test systems with different methods are shown in Table 6. As shown in this table, using the FTSLM can reduce the execution time in comparison with other methods. As an example, using FTSLM to solve the power flow problem in the 43-bus test system decreases the execution time about 39.5%, 60% and 35.2% in comparison with LM, NRJM and TSLM methods, respectively. Note that by increasing the size of the power system, this difference becomes more apparent. For example, in 2383-bus test system, the CPU time of the FTSLM method is about 45% of the TSLM method.

Note that the IRN method has diverged in all of the case studies and the NRJM and LM methods have diverged in 2383-bus power system.

Figs. 9–13 illustrate comparison between the convergence error of the NRJM, LM, TSLM methods and the proposed FTSLM method.

As shown in these figures, using the FTSLM can decrease the residual error in less iterations in comparison with other methods. For example, in 118-bus test system, the NRJM, LM, TSLM and proposed FTSLM methods have converged in 6, 8, 6 and 3 iterations, respectively.

Note that the LM method is second-order convergent while the TSLM is fourth-order convergent. However, in the small cases such as 11-bus and 13-bus test systems, the performance of the TSLM is not obvious (Table 6-rows 2–3, Figs. 9 and 10). The main reason for this is that the TSLM uses three Jacobian matrices and three inversions in each iteration and consumed time for these processes is not negligible in comparison with other parts of the problem solution. However, in the large systems (i.e. 118-bus and 2383-bus systems) the advantages of the TSLM, in comparison with the LM method, will appear and cause to decrease CPU times and number of iterations (Table 6-rows 5–6, Figs. 12 and 13).

7. Relationship between voltage instability and ill-conditioned power system

A power system enters to the voltage instability state when a change in topology of the system or load increase, causes uncontrollable voltage degradation [45]. In case of the load increment, eigenvalues of the Jacobian matrix will change and condition of the power system may change to an ill state [33]. To evaluate the effectiveness of the proposed FTSLM method, we have changed the loading condition of the 11-bus and 43-bus test systems and solved the power flow problem. Eqs. (30) and (31) present the loading condition (active and reactive demand) of the buses.

$$\boldsymbol{P_d} = \rho \boldsymbol{P_{d_0}} \tag{30}$$

$$\boldsymbol{Q_d} = \rho \boldsymbol{Q_{d_0}} \tag{31}$$

where $\boldsymbol{Q_{d_0}}$ and $\boldsymbol{P_{d_0}}$ are initial vectors of the reactive and active demands of the buses, respectively. Also, parameter ρ changes the loading condition of the test systems [33]. Increasing of the loading condition causes to increase the condition number of the systems (Table 7-columns: ρ and N). As shown in Table 7, despite of increasing the loading condition of the systems and approaching the voltage instability state, the proposed FTSLM can solve the power flow problem in the ill-conditioned test systems.

8. Conclusions

In this paper, a FLT based TSLM method has been proposed to solve the power flow equations of the ill-conditioned power systems. The re-

Table 6
CPU time and number of iterations of the different methods.

Test system	Conventional methods						TSLM method		FTSLM method	
	IRN method [41]		NRJM method [43,44]		LM method [42]					
	CPU time (s)	Number of iteration	CPU time (s)	Number of iteration	CPU time (s)	Number of iteration	CPU time (s)	Number of iteration	CPU Time (s)	Number of iteration
11-bus	Div.	Div.	0.213	3	0.121	5	0.133	5	0.091	2
13-bus	Div.	Div.	0.160	4	0.081	6	0.117	4	0.057	2
43-bus	Div.	Div.	0.390	6	0.258	8	0.241	6	0.156	3
118-bus	Div.	Div.	0.842	5	0.880	10	0.687	7	0.294	3
2383-bus	Div.	Div.	–	Div.	Div.	–	441.72	11	200.780	5

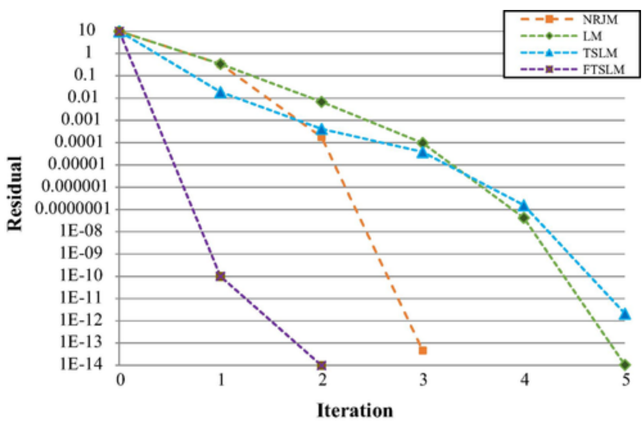

Fig. 9. Residual error in the 11-bus ill-conditioned power system.

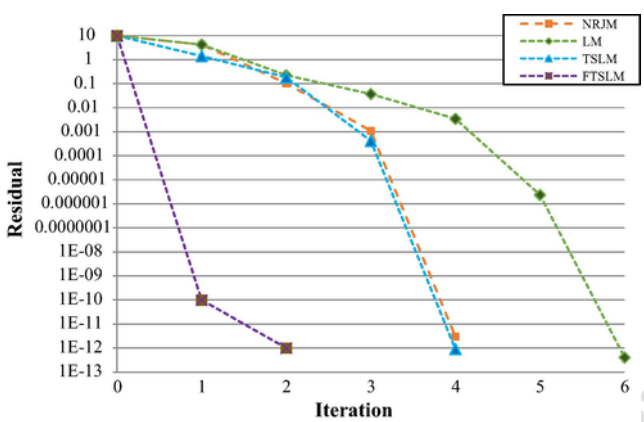

Fig. 10. Residual error in the 13-bus ill-conditioned distribution system.

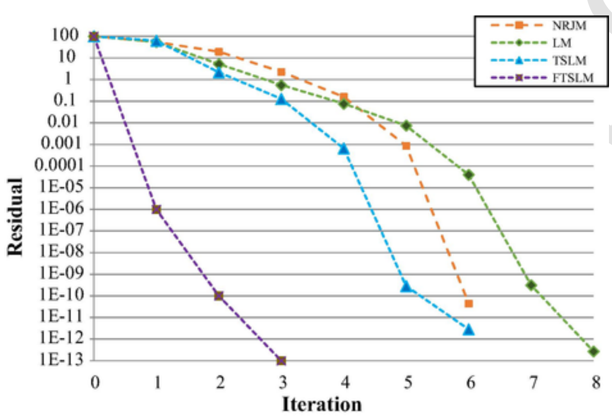

Fig. 11. Residual error in the 43-bus ill-conditioned power system.

lated formulations and algorithms have been presented and applied to 11-bus, 13-bus, 43-bus, 118-bus and 2383-bus test power systems. The inputs to the FIS are reactive and real power mismatches ($\Delta S_k^{(1)}$, $\Delta S_k^{(2)}$ and $\Delta S_k^{(3)}$), while the outputs are phase and magnitude of the voltages of the buses.

The simulation results show that the new proposed FTSLM method is very fast and reliable to use in the ill-conditioned power systems. The number of iterations and CPU times in the proposed method are lower than the benchmark methods (IRNM, LM, NRJM and TSLM methods) (Figs. 9–13 and Tables 6 and 7). The main capabilities and features of the FTSLM method are concluded as follows:

- Simple structure

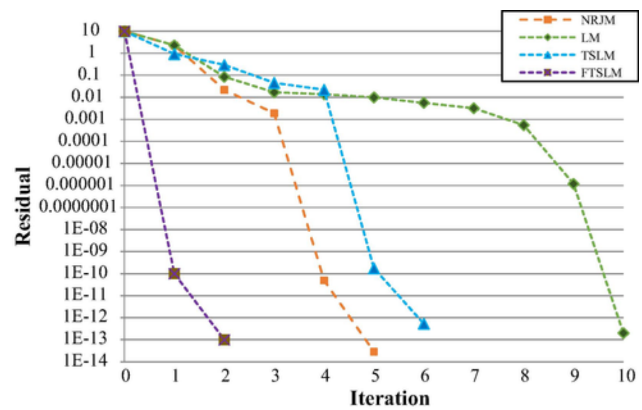

Fig. 12. Residual error in the 118-bus ill-conditioned power system.

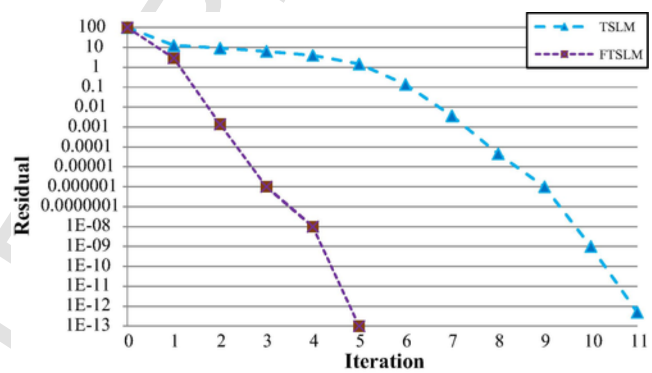

Fig. 13. Residual error in the 2383-bus ill-conditioned power system.

- Fast convergence
- Low number of iterations
- Less computation time

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Appendix A. Proof of the fourth order convergence of the TSLM method

In this section, proof of the fourth order convergence of the TSLM method based on [38] is presented. By using "singular value decomposition" to decompose the Jacobian matrix ($\boldsymbol{J_k}$ in Eq. (8)):

$$J_k = U_1 \sum_1 V_1^T + U_2 \sum_2 V_2^T \tag{32}$$

Also, parameters d_k^{LM}, d_k^{MLM} and d_k^{HLM} can be computed as follows [38]:

$$\begin{aligned} d_k^{LM} = & -V_1 \left(\sum{}_1^2 + \lambda_k I \right)^{-1} \sum_1 U_1^T F_k \\ & - V_2 \left(\sum{}_2^2 + \lambda_k I \right)^{-1} \sum_2 U_2^T F_k \end{aligned} \tag{33}$$

Table 7
CPU time for 11-bus and 43-bus in different loading conditions.

Loading condition (ρ)	11-Bus				43-Bus			
	π_{min}	π_{max}	*CN*	CPU time (s)	π_{min}	π_{max}	*CN*	CPU time (s)
0.9	0.1131	122.20	$0.1080*10^4$	0.0870	$0.9495*10^{-1}$	$0.2428*10^4$	$0.2557*10^5$	0.1490
0.95	0.1128	122.16	$0.1082*10^4$	0.0905	$0.9484*10^{-1}$	$0.2426*10^4$	$0.2558*10^5$	0.1541
1.00	0.1126	122.10	$0.1086*10^4$	0.0910	$0.9476*10^{-1}$	$0.2426*10^4$	$0.2560*10^5$	0.1560
1.05	0.1122	122.08	$0.1088*10^4$	0.0925	$0.9462*10^{-1}$	$0.2423*10^4$	$0.2561*10^5$	0.1575
1.10	0.1120	122.05	$0.1089*10^4$	0.0936	$0.9451*10^{-1}$	$0.2421*10^4$	$0.2562*10^5$	0.1599
1.15	0.1116	122.01	$0.1093*10^4$	0.0939	$0.9440*10^{-1}$	$0.2419*10^4$	$0.2562*10^5$	0.1606
1.20	0.1114	121.97	$0.1094*10^4$	0.0944	$0.9430*10^{-1}$	$0.2417*10^4$	$0.2563*10^5$	0.1622
1.25	0.1110	121.93	$0.1098*10^4$	0.0945	$0.9416*10^{-1}$	$0.2415*10^4$	$0.2565*10^5$	0.1663

$$d_k^{MLM} = -V_1\left(\sum{}_1^2 + \lambda_k I\right)^{-1}\sum_1 U_1^T F(y_k) - V_2\left(\sum{}_2^2 + \lambda_k I\right)^{-1}\sum_2 U_2^T F(y_k) \tag{34}$$

$$d_k^{HLM} = -V_1\left(\sum{}_1^2 + \lambda_k I\right)^{-1}\sum_1 U_1^T F(z_k) - V_2\left(\sum{}_2^2 + \lambda_k I\right)^{-1}\sum_2 U_2^T F(z_k) \tag{35}$$

So:

$$\begin{aligned} F_k + J_k d_k^{LM} &= F_k - U_1\sum_1\left(\sum{}_1^2 + \lambda_k I\right)^{-1}\sum_1 U_1^T F_k - U_2\sum_2\left(\sum{}_2^2 + \lambda_k I\right)^{-1}\sum_2 U_2^T F_k \\ &= \lambda_k U_1\left(\sum{}_1^2 + \lambda_k I\right)^{-1} U_1^T F_k - \lambda_k U_2\sum_2\left(\sum{}_2^2 + \lambda_k I\right)^{-1} U_2^T F_k + U_3 U_3^T F_k \end{aligned} \tag{36}$$

$$F(y_k) + J_k d_k^{MLM} = F(y_k) - U_1\Sigma_1\left(\Sigma{}_1^2 + \lambda_k I\right)^{-1}\Sigma_1 U_1^T F(y_k)\cdot U_2\Sigma_2\left(\Sigma{}_2^2 + \lambda_k I\right)^{-1}\Sigma_2 U_2^T F(y_k)$$

$$F(z_k) + J_k d_k^{HLM} = F(z_k) - U_1\Sigma_1\left(\Sigma{}_1^2 + \lambda_k I\right)^{-1}\Sigma_1 U_1^T F(z_k) - U_2\Sigma_2\left(\Sigma{}_2^2 + \lambda_k I\right)^{-1}\Sigma_2 U_2^T F(z_k)$$

By using three proofed lemmas in [38] and also considering theory of the matrix perturbation and the Lipschitzness of $\boldsymbol{J_k}$:

$$\left\| diag(\Sigma_1 - \overline{\Sigma_1}, \Sigma_1, 0)\right\| \leqslant \left\|\boldsymbol{J_k} - \overline{\boldsymbol{J_k}}\right\| \leqslant L_1\left\|\boldsymbol{x_k} - \overline{\boldsymbol{x_k}}\right\| \tag{39}$$

By considering Eq. (39):

$$\|\Sigma_1\|^{-1} = \left|\frac{1}{\sigma_r}\right| \leqslant \left|\frac{1}{\overline{\sigma_r} - L_1\|\boldsymbol{x_k} - \overline{\boldsymbol{x_k}}\|}\right| \tag{40}$$

this denotes:

$$\|\Sigma_1\|^{-1} \leqslant \left|\frac{2}{\overline{\sigma_r}}\right| \tag{41}$$

By using three above-mentioned lemmas [38]:

$$\left\| d_{k+1}^{LM} + \eta_{k+1} d_{k+1}^{MLM} + d_{k+1}^{HLM}\right\| \leqslant O\left\| d_{k+1}^{LM} + \eta_{k+1} d_{k+1}^{MLM} + d_{k+1}^{HLM}\right\|^4 \tag{42}$$

which means that $\{x_{(k)}\}$ (in Eqs. (7)–(9)) achieved by the TSLM method converges to the resolution set X^* with fourth order [38].

Appendix B. Proof of the convergence of the proposed FTSLM method

Give two fuzzy sets $\tilde{\Delta} s = \left[\Delta\tilde{S}_k^{(1)}, \Delta\tilde{S}_k^{(2)}, \Delta\tilde{S}_k^{(3)}\right]$ and $\tilde{\varphi} = \left[\tilde{\varphi}_k^{LM}, \tilde{\varphi}_k^{MLM}, \tilde{\varphi}_k^{HLM}\right]$ (see Eq. (25)) be defined in universe of discourse (U) as:

$$\Delta\tilde{S}_k^{(1)} = \sum_{\sigma\in\Omega}\mu_{\Delta\tilde{S}_k^{(1)}}(\sigma)/\sigma \tag{43}$$

$$\Delta\tilde{S}_k^{(2)} = \sum_{\sigma\in\Omega}\mu_{\Delta\tilde{S}_k^{(2)}}(\sigma)/\sigma \tag{44}$$

$$\Delta\tilde{S}_k^{(3)} = \sum_{\sigma\in\Omega}\mu_{\Delta\tilde{S}_k^{(3)}}(\sigma)/\sigma \tag{45}$$

and,

$$\tilde{\varphi}_k^{LM} = \sum_{\sigma\in\Omega}\mu_{\tilde{\varphi}_k^{LM}}(\sigma)/\sigma \tag{46}$$

$$\widetilde{\varphi_k^{MLM}} = \sum_{\sigma\in\Omega} \mu_{\widetilde{\varphi_k^{MLM}}}(\sigma)/\sigma \tag{47}$$

$$\widetilde{\varphi_k^{HLM}} = \sum_{\sigma\in\Omega} \mu_{\widetilde{\varphi_k^{HLM}}}(\sigma)/\sigma \tag{48}$$

where $\mu_{\widetilde{\Delta s}} = \sum_{u\in J_\sigma^{\Delta s}} f_\sigma(u)/u$ and $\mu_{\widetilde{\varphi}} = \sum_{w\in J_\sigma^{\varphi}} g_\sigma(w)/w$. Based on Zadeh's extension theory, the membership grades for union and intersection of $\widetilde{\Delta s}$ and $\widetilde{\varphi}$ would be:

$$\begin{aligned}\mu_{\widetilde{\Delta s}\cup\widetilde{\varphi}}(\sigma) &= \sum_{u\in J_\sigma^{\Delta s}} (f_\sigma(u) \wedge g_\sigma(w))/(u\vee w) \\ &\equiv \mu_{\widetilde{\Delta s}}(\sigma)\sqcup\mu_{\widetilde{\varphi}}(\sigma)\end{aligned} \tag{49}$$

$$\begin{aligned}\mu_{\widetilde{\Delta s}\cap\widetilde{\varphi}}(\sigma) &= \sum_{u\in J_\sigma^{\Delta s}} (f_\sigma(u) \wedge g_\sigma(w))/(u \wedge w) \\ &\equiv \mu_{\widetilde{\Delta s}}(\sigma)\sqcap\mu_{\widetilde{\varphi}}(\sigma)\end{aligned} \tag{50}$$

where $\sqcup$ and $\sqcap$ denote join and meet, respectively. Also, $\vee$ and $\wedge$ denote max t-conorm and min t-norm, respectively. Note that, $\mu_{\widetilde{\Delta s}}(\sigma)$ and $\mu_{\widetilde{\varphi}}(\sigma)$ are type-1 fuzzy sets [46].

In order to proof the convergence of the proposed FTSLM method, Given $F_1 = \sum_{u\in U} f(u)/u$ and $F_2 = \sum_{w\in U} g(w)/w$ be two convex of fuzzy sets defined over U such that their maximum (or minimum) degree of intersection is attained at Z (σ = Z). In the other words, $\underset{\sigma}{Supremum}(f(\sigma) \wedge g(\sigma)) = Y$ attained at σ = Z, and $\underset{\sigma}{Supremum}(f(\sigma)) = h_f$ attained at $\sigma = \xi_f$, and $\underset{\sigma}{Supremum}(g(\sigma)) = h_g$ at $\sigma = \xi_g$, then Z will be between ξ_f and ξ_g i.e., either ξ_g?Z?ξ_f or ξ_f?Z?ξ_g [46].

By considering both ξ_f and ξ_g are on the same side of Z i.e. $\xi_g <$ Z, $\xi_f <$ Z and supposing ξ_f?ξ_g and also, $\mathrm{Z} = \underset{f(\theta)\wedge g(\theta)=Y}{Infimum}(\theta)$:

$$\exists\lambda,\lambda' \in [0,1] | \lambda\xi_f + (1-\lambda)\mathrm{Z} = \theta \in (\xi_g, \mathrm{Z}) \tag{51}$$

So $f(\theta) \geqslant Min(f(\xi_f), f(\mathrm{Z})) \rightarrow f(\theta) \geqslant Y$ and $g(\theta) \geqslant Min(g(\xi_g), g(\mathrm{Z})) \rightarrow g(\theta) \geqslant Y$ [46]. This means that $f(\theta) \wedge g(\theta) \geqslant Y$, which is contradiction. So it can be concluded that Z is located between ξ_f and ξ_g i.e. either $\xi_g <$ Z $< \xi_f$ or $\xi_f <$ Z $< \xi_g$. The proof is completed.

References

[1] W.F. Tinney, C.E. Hart, Power flow solution by Newton's method, IEEE Trans Power Apparat Syst 86 (11) (1967) 1449–1460.
[2] S. Iwamoto, Y. Tamura, A fast load flow method retaining nonlinearity, IEEE Trans Power App Syst 97 (5) (1978) 1587–1599.
[3] B. Scott, Review of load flow calculation methods, IEEE Proc 62 (7) (1974) 916–929.
[4] B. Stott, O. Alsac, Fast decoupled load flow, IEEE Trans Power Apparat Syst 93 (3) (1974) 859–869.
[5] V. Ajjarapu, C. Christy, The continuation power flow: a tool for steady state voltage stability analysis, IEEE Trans Power Syst 7 (1) (1992) 416–423.
[6] Y. Chen, C.A. Shen, Jacobian-free Newton-GMRES(m) method with adaptive preconditioner and its application for power flow calculations, IEEE Trans Power Syst 21 (3) (2006) 1096–1103.
[7] Y.S. Zhang, H.D. Chiang, Fast Newton-FGMRES Solver for large-scale power flow study, IEEE Trans Power Syst 25 (2) (2010) 769–776.
[8] S. Iwamoto, Y. Tamura, A load flow calculation method for ill-conditioned power systems, IEEE Trans Power Apparat Syst 100 (4) (1981) 1736–1743.
[9] S.C. Tripathy, G.D. Prasad, O.P. Malik, G.S. Hope, Load flow solutions for ill-conditioned power systems by a Newton-like method, IEEE Trans Power Apparat Syst 101 (10) (1982) 3648–3657.
[10] F. Milano, Continuous Newton's method for power flow analysis, IEEE Trans Power Syst 24 (1) (2009) 50–57.
[11] R. Pourbagher, S.Y. Derakhshandeh, Application of high-order Levenberg Marquardt method for solving the power flow problem in the ill-conditioned systems, IET Gener Transm Distrib 10 (12) (2016) 3017–3022.
[12] A. Shahriari, H. Mokhlis, M. Karimi, A.H.A. Bakar, H.A. Illias, Quadratic discriminant index for optimal multiplier load flow method in ill conditioned system, Int J Electr Power Energy Syst 60 (1) (2014) 378–388.
[13] R. Pourbagher, S.Y. Derakhshandeh, A powerful method for solving the power flow problem in the ill-conditioned systems, Int J Electr Power Energy Syst 94 (1) (2018) 88–96.
[14] H. Chen, X. Luo, Y. Li, H. Fu, X. Zheng, X. Yuan, Ill-conditioned load flow study based on asynchronous parallel computing in rural power distribution networks, J Converg Inform Technol (JCIT) 7 (19) (2012) 261–268.
[15] X. Yang, X. Zhou, Application of asymptotic numerical method with homotopy techniques to power flow problems, Int J Electr Power Energy Syst 57 (1) (2014) 375–383.
[16] L.A. Zadeh, Fuzzy sets. Inf, Control 8 (1965) 338–353.
[17] Chow MY. Fuzzy systems. In: CRC Press Ind Electronics Handbook; 1996.
[18] J.A. Momoh, X.W. Ma, K. Tomsovic, Overview and literature survey of fuzzy set theory in power system, IEEE Trans Power Syst 10 (3) (1995) 1676–1690.
[19] M.Y. Chow, H. Tram, Applications of fuzzy logic technology for spactial load forecasting, IEEE Trans Power Syst 12 (3) (1997) 1360–1366.
[20] H. Mori, H. Kobayashi, Optimal fuzzy inference for short-term load forecasting, IEEE Trans Power Syst 11 (1) (1996) 390–396.
[21] V. Miranda, J.T. Saraiva, Fuzzy modeling of power system optimal power flow, IEEE Trans Power Syst 7 (1992) 843–849.
[22] Miranda V, Matos M. Distribution system planning with fuzzy models and techniques. In: Proc CIRED—10th Int Conf Elect Dist Brighton U.K.; 1989. p. 472–6.
[23] Z. Wang, F.L. Alvarado, Interval arithmetic in power flow analysis, IEEE Trans Power Syst 7 (3) (1992) 1341–1349.
[24] A. Kaufmann, M.M. Gupta, Fuzzy mathematical models in engineering and management science, North-Holland, Amsterdam, 1988.
[25] Miranda V, Matos MA, Saraiva JT. Fuzzy load flow-new algorithms incorporating uncertain generation and load representation. In: Proceedings of the 10th power syst. comput. conf., Graz, Austria; 1990. p. 621–7.
[26] K.L. Lo, Y.J. Lin, W.H. Siew, Fuzzy-logic method for adjustment of variable parameter in load-flow calculation, IEE Proc Gen Trans Distr 146 (3) (1999) 276–282.
[27] J.G. Vlachogiannis, Fuzzy logic application in load flow studies, IEE Proc Gen Trans Distr 148 (1) (2001) 34–40.
[28] Dixit S, Srivastava L, Agnihotri G. Power flow analysis using fuzzy logic. In: IEEE Power India Conference; 2006. p. 1–7.
[29] E.M. Gouveia, M.A. Matos, Symmetric AC fuzzy power flow model, Eur. J. Oper. Res. 197 (3) (2009) 1012–1018.
[30] M. Helenoa, J. Sumaili, J. Meirinhos, M.A. Rosa, A linearized approach to the symmetric fuzzy power flow for the application to real systems, Electr Power nd Energy Syst. 54 (1) (2014) 610–618.
[31] B.M. Kalesar, A. Seifi, Fuzzy load flow in balanced and unbalanced radial distribution systems incorporating composite load model, Electr Power Energy Syst 32 (1) (2010) 17–23.
[32] H. Wu, L. Cai, A new approach combing connection number and fuzzy simulation to calculating power flow of distribution network considering uncertainty, Electr Power Energy Syst 53 (1) (2013) 884–892.
[33] S.Y. Derakhshandeh, R. Pourbagher, Application of high-order Newton-like methods to solve power flow equations, IET Gener Transm Distrib 10 (8) (2016) 1853–1859.
[34] B.T. Chao, H.L. Li, E.J. Scott, On the solution of ill-conditioned, simultaneous, linear, algebraic equations by machine computation, University of Illinois, Urbana-Champaign, 1961.
[35] R. Pourbagher, S.Y. Derakhshandeh, A powerful method for solving the power flow problem in the ill-conditioned systems, Electr Power Energy Syst 94 (1) (2018) 88–96.
[36] H.B. Keller, Lectures on numerical methods in bifurcation problems, Springer-Verlag, Berlin, 1987.
[37] A. Griewank, On solving nonlinear equations with simple singularities or nearly singular solutions, SIAM Rev 27 (4) (1985) 537–563.
[38] L. Chen, A high-order modified Levenberg–Marquardt method for systems of nonlinear equations with fourth-order convergence, Appl Math Comput 285 (2016) 79–93.
[39] R.D. Zimmerman, C.E. Murillo-Sánchez, R.J. Thomas, Matpower: steady-state operations, planning, and analysis tools for power systems research and education, IEEE Trans Power Syst 26 (1) (2011) 12–19.
[40] A. Shahriari, H. Mokhlis, A.H.A. Bakar, M. Karimi, M.M. Aman, The non-divergence Newton-based load flow method in ill conditioned system, Int J Sci Eng Res 3 (6) (2012) 1–6.
[41] A. Bakushinsky, A. Goncharsky, Ill-posed problems: theory and applications, Kluwer Academic Publishers, 1994.
[42] N.D. Rao, S.C. Tripathy, Power system static state estimation by the Levenberg Marquardt algorithm, IEEE Trans Power App Syst 99 (2) (1980) 695–702.
[43] Lagace PJ, Vuong MH, Kamwa I. Improving power flow convergence by Newton Raphson with a Levenberg-Marquardt method. In: IEEE power and energy society general meeting - conversion and delivery of electrical energy in the 21st century conference Pittsburgh PA; 2008. p. 1–6.
[44] Lagace PJ. Power flow methods for improving convergence. In: IECON 2012–38th annual conference on IEEE industrial electronics society; 2012. p. 1387–92.
[45] M. Eremia, M. Shahidehpour, Handbook of electrical power system dynamics: modeling, stability, and control, The Institute of Electrical and Electronics Engineers Inc, 2013.
[46] Tahayori H, Antoni GD. A simple method for performing type-2 fuzzy set operations based on highest degree of intersection hyperplane. Fuzzy Information Processing Society, 2007. NAFIPS '07. Annual Meeting of the North American San Diego, CA, USA; 2007. p. 404–9.