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On Some Linear Operators Preserving Disjoint Support Property

Noha Eftekhari[1] * and Ali Bayati Eshkaftaki[2]

ABSTRACT. The aim of this work is to characterize all bounded linear operators $T : \ell^p(I) \to \ell^p(I)$ which preserve disjoint support property. We show that the constant coefficients of all isometries on $\ell^p(I)$ are in the class of such operators, where $2 \neq p \in [1, \infty)$ and I is a non-empty set. We extend preserving disjoint support property to linear operators on $\mathfrak{c}_0(I)$. At the end, we obtain some equivalent properties of isometries on Banach spaces.

1. Introduction and preliminaries

Let $(X, \sum, \mu)$ be a measure space. For $1 \leq p < \infty$, we consider $L^p(\mu)$ consisting of all $\sum$-measurable functions $f : X \to \mathbb{R}$ with the finite norm defined by

$$\|f\|_p = \left(\int_X |f(x)|^p \mathrm{d}\mu(x) \right)^{\frac{1}{p}}.$$

In 2005, N.L. Carothers [6] considered disjoint support property in the following sense and obtained Theorems 1.2 and 1.3.

Definition 1.1. Two elements $f, g \in L^p(\mu)$ are called disjointly supported if $f(x)g(x) = 0$ for μ almost every x. Whenever f and g are disjointly supported, we denote simply $fg = 0$.

In $\ell^p := \ell^p(\mathbb{N})$ or $\mathfrak{c}_0$, the Banach space of all real sequences vanishing at infinity with the supremum norm, two elements $x = (x_n)$ and $y = (y_n)$ are disjointly supported, if $x_n y_n = 0$ for all $n \in \mathbb{N}$.

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* Corresponding author.

Theorem 1.2 ([6]). *Let $1 \leq p < \infty$, $p \neq 2$, and $f, g \in L^p(\mu)$. Then the following conditions are equivalent.*

(i) *f and g are disjointly supported.*
(ii) *For all $\alpha, \beta \in \mathbb{F}$, $\|\alpha f + \beta g\|^p = \|\alpha f\|^p + \|\beta g\|^p$.*
(iii) *$\|f + g\|^p + \|f - g\|^p = 2\big(\|f\|^p + \|g\|^p\big)$.*

Theorem 1.3 ([6]). *Let $T : L^p(\mu) \to L^p(\mu)$ be an isometry, where $2 \neq p \in [1, \infty)$. Then $fg = 0$ if and only if $TfTg = 0$.*

The study of disjoint support preserving operators began in 1932 by S. Banach [5] (although not explicitly) and it has been widely developed since then not only in the commutative but also in the noncommutative setting. In many cases, disjointness preserving operators are identified with weighted composition operators, and they are closely related to isometries.

For a survey of disjoint support property, we refer the reader to the classical book [6] by Carothers and [14]. Many mathematicians improved this notion. Abramovich et al. considered disjointness preserving operators in [1, 2].

In 1990, J.T. Chan [7] studied operators with the disjoint support property in the following sense. A bounded linear operator T on $C(K, X)$ is said to have the disjoint support property if for any $f, g \in C(K, X)$ such that $\|f(k)\| \, \|g(k)\| = 0$ for all $k \in K$, we always have

$$\|Tf(k)\| \|Tg(k)\| = 0,$$

for all $k \in K$, where K is a compact Hausdorff space and X is a real or complex Banach space and $C(K, X)$ is the Banach space of all continuous X-valued functions defined on K with the supremum norm. If T has this property, some authors called T a separating map, for example, see [9, 11]. In this work, we consider the operators with disjoint support property in the sense of Definition 2.3. Also in 2009, H. Zhang [15] investigated reproducing kernel Hilbert spaces (RKHS) where two functions are orthogonal whenever they are disjointly supported. In this work, we characterize all bounded linear operators on $\ell^p(I)$ and $\mathfrak{c}_0(I)$ which preserve disjoint support property and obtain some properties of them, where I is a non-empty set.

From now on, I is a non-empty set and $\mathrm{e}_i : I \to \mathbb{R}$ is defined by $\mathrm{e}_i(j) = \delta_{ij}$, the Kronecker delta, for all $j \in I$.

We organize this paper as follows. Section 2 is devoted to considering linear operators preserving disjoint support property on $\ell^p(I)$ and derive some interesting equivalent conditions on $T : \ell^p(I) \to \ell^p(I)$ to preserve disjoint support. We show that all columns of the matrix form of the operator $T : \ell^p \to \ell^p$ which $\|Tx\| = \|T\| \, \|x\|$ are in $\mathfrak{c}_0$. Some examples of preserving disjoint support linear operators on ℓ^p are presented. Also, we develop the theory of preserving disjoint support to bounded linear operators $T : \mathfrak{c}_0(I) \to \mathfrak{c}_0(I)$ and obtain some similar theorems.

2. Operators preserving disjoint support property on $\ell^p(I)$ and $\mathfrak{c}_0(I)$

The topic of linear preservers is of interest to a large group of matrix theorists. For a survey of linear preserver problems see [13], and for relative papers and books in the theory of majorization see [3, 4].

Throughout this section, I is a non-empty set, $p \in [1, +\infty)$, and $\ell^p(I)$ is the Banach space of all functions $f : I \to \mathbb{R}$ with the finite norm defined by

$$\|f\|_p = \left(\sum_{j \in I} |f(j)|^p \right)^{\frac{1}{p}}.$$

Let $T : \ell^p(I) \to \ell^p(I)$ be a bounded linear operator. An easy computation shows that, T is represented by a (finite or infinite) matrix $(t_{ij})_{i,j \in I}$ in the sense that

$$(Tf)(i) = \sum_{j \in I} t_{ij} f(j), \quad (f \in \ell^p(I), i \in I),$$

where $t_{ij} = (T\mathrm{e}_j)(i)$, and $\mathrm{e}_j : I \to \mathbb{R}$ is defined by $\mathrm{e}_j(i) = \delta_{ij}$, the Kronecker delta. We can incorporate T to its matrix form $(t_{ij})_{i,j \in I}$.

For the Banach space X, let $\mathcal{B}(X)$ be the set of all bounded linear operators on X. We define $\mathfrak{I}(X)$ and $\mathfrak{Iso}(X)$ as follows:

$$\begin{aligned} \mathfrak{I}(X) &= \{T \in \mathcal{B}(X); \|Tx\| = \|T\| \, \|x\| \text{ for all } x \in X\}, \\ \mathfrak{Iso}(X) &= \{T \in \mathcal{B}(X); T \text{ is an isometry } \}. \end{aligned}$$

Remark 2.1. Let $T \in \mathfrak{I}(\ell^p(I))$. As $\|\mathrm{e}_i\| = 1$, we have $\|T\mathrm{e}_i\| = \|T\|$ for all $i \in I$. Also, for all $m, j \in I$ such that $|(T\mathrm{e}_j)(m)| = \|T\|$, we have $(T\mathrm{e}_j)(i) = 0$ for $i \neq m$. Because by using the definition of the isometry T we get

$$\begin{aligned} \|T\|^p &= \|T\mathrm{e}_j\|^p \\ &= \sum_{i \in I} |(T\mathrm{e}_j)(i)|^p \\ &\geq |(T\mathrm{e}_j)(m)|^p \\ &= \|T\|^p, \end{aligned}$$

so it has to be $(T\mathrm{e}_j)(i) = 0$ for $i \neq m$ and we get

$$\sum_{i \in I} |(T\mathrm{e}_j)(i)|^p = \|T\|^p.$$

Let $T \in \mathfrak{I}(\ell^p)$. Then all columns of T are in $\mathfrak{c}_0$. Since $\mathrm{e}_j \in \ell^p$, hence the column $T\mathrm{e}_j$ is surely contained in codomain ℓ^p which is a subset of $\mathfrak{c}_0$.

An isometry is a transformation which preserves the distance between the elements of a space. In 1932, Banach [5] showed that every linear isometry on the space of continuous real-valued functions on a compact metric space must transform a continuous function f into a continuous function g satisfying

$$g(t) = h(t)f(\varphi(t)),$$

where $\|h\| = 1$ and φ is a homeomorphism. Also, R.J. Fleming and J.E. Jamison [8] investigated isometries on Banach spaces.

Remark 2.2. Let X be a Banach space. For a nonzero $T \in \mathcal{B}(X)$ and $x, y \in X$, the following conditions are equivalent.

(i) $T \in \mathfrak{I}(X)$.
(ii) $\frac{T}{\|T\|} \in \mathfrak{Iso}(X)$.
(iii) T is a norm-preserving, (i.e. $\|x\| = \|y\|$ implies $\|Tx\| = \|Ty\|$).
(iv) $\|x\| \leq \|y\|$ implies $\|Tx\| \leq \|Ty\|$.
(v) $\|Tx\| = \|Ty\|$ implies $\|x\| = \|y\|$.
(vi) $\|Tx\| \leq \|Ty\|$ implies $\|x\| \leq \|y\|$.

In the following, we define bounded linear operator with preserving disjoint support property.

Definition 2.3. Let $T : \ell^p(I) \to \ell^p(I)$ be a bounded linear operator. We call T preserving disjoint support property, if for all $f, g \in \ell^p(I)$ the equality $fg = 0$ implies $TfTg = 0$.

Let $\mathcal{P}_{ds}(\ell^p(I))$ denote the set of all bounded linear operators $T : \ell^p(I) \to \ell^p(I)$ which preserve disjoint support property on $\ell^p(I)$.

Remark 2.4. Some general properties of $\mathcal{P}_{ds}(\ell^p(I))$ are as follows.

- $0, \mathrm{id} \in \mathcal{P}_{ds}(\ell^p(I))$.
- If $T_1, T_2 \in \mathcal{P}_{ds}(\ell^p(I))$, then $T_1 \circ T_2 \in \mathcal{P}_{ds}(\ell^p(I))$.
- If $T \in \mathcal{P}_{ds}(\ell^p(I))$, then $\lambda T \in \mathcal{P}_{ds}(\ell^p(I))$ for all $\lambda \in \mathbb{R}$.
- Any constant coefficient of a permutation on $\ell^p(I)$ lies in $\mathcal{P}_{ds}(\ell^p(I))$.

Example 2.5. Let $a, b \in \mathbb{R}$ and $T : \ell^p \to \ell^p$ be defined by

$$Tf = (af_1, bf_1, af_2, bf_2, \ldots),$$

for $f = (f_1, f_2, \ldots) \in \ell^p$. Clearly $T \in \mathcal{P}_{ds}(\ell^p)$.
In general, let (n_k) be a bounded sequence in $\mathbb{N}$. Then the operator $T : \ell^p \to \ell^p$ defined by

$$Tf = (\underbrace{af_1, \ldots, af_1}_{n_1}, \underbrace{bf_1, \ldots, bf_1}_{n_2}, \underbrace{af_2, \ldots, af_2}_{n_3}, \underbrace{bf_2, \ldots, bf_2}_{n_4}, \ldots),$$

for $f = (f_1, f_2, \ldots) \in \ell^p$ lies in $\mathcal{P}_{ds}(\ell^p)$.

In the next theorem, we consider equivalent conditions for $T : \ell^p(I) \to \ell^p(I)$ to preserve disjoint support.

Theorem 2.6. *Let $T : \ell^p(I) \to \ell^p(I)$ be a bounded linear operator. Then the following conditions are equivalent.*

(i) $T \in \mathcal{P}_{ds}(\ell^p(I))$.

(ii) *For all distinct $j, j' \in I$, $T\mathrm{e}_j T\mathrm{e}_{j'} = 0$.*

(iii) *There exists a uniformly bounded family of disjointly supported $\{u_j; j \in I\}$ in $\ell^p(I)$ such that*

$$Tf = \sum_{j \in I} f(j) u_j \quad \textit{for all} \quad f \in \ell^p(I).$$

Proof(i) $\Rightarrow$ (ii) It is obvious.

(ii) $\Rightarrow$ (iii) For all $i \in I$, let $u_i = T\mathrm{e}_i$. Then for all $f \in \ell^p(I)$, we have

$$\begin{aligned} Tf &= T\left(\sum_{j \in I} f(j)\mathrm{e}_j\right) \\ &= \sum_{j \in I} f(j) T\mathrm{e}_j \\ &= \sum_{j \in I} f(j) u_j. \end{aligned}$$

(iii) $\Rightarrow$ (ii) For any distinct $j, j' \in I$, we have $T\mathrm{e}_j T\mathrm{e}_{j'} = u_j u_{j'} = 0$.

(ii) $\Rightarrow$ (i) Suppose that $f, g \in \ell^p(I)$ are disjointly supported. For any given $i \in I$,

$$\begin{aligned} (Tf)(i)(Tg)(i) &= \sum_{j \in I} f(j) T\mathrm{e}_j(i) \sum_{j' \in I} g(j') T\mathrm{e}_{j'}(i) \\ &= \sum_{j \in I} \sum_{j' \in I} f(j) g(j') T\mathrm{e}_j(i) T\mathrm{e}_{j'}(i) \\ &= 0. \end{aligned}$$

The last equality holds, since whenever $j = j'$, it follows that $f(j)g(j') = 0$ and if $j \neq j'$, we have $T\mathrm{e}_j(i) T\mathrm{e}_{j'}(i) = 0$.

□

The elements of $\mathfrak{I}(X)$ for $X = L_p, \ldots$ have been considered in [12, Theorem 3.1] and in [8]. Now in the next theorem, we characterize the elements of $\mathfrak{I}(X)$ for discrete case $X = \ell^p(I)$.

Theorem 2.7. *(Characterization theorem). Let $p \in [1, \infty)$, $p \neq 2$, $f, g \in \ell^p(I)$, and $T : \ell^p(I) \to \ell^p(I)$ be a bounded linear operator. Then the following conditions are equivalent.*

(i) $T \in \mathfrak{I}(\ell^p(I))$.

(ii) *If $fg = 0$, then $TfTg = 0$. Furthermore, $\|T\mathrm{e}_i\|$ is constant independent of $i \in I$.*
(iii) *There exists a family of disjointly supported $\{u_j;\ j \in I\}$ with a constant norm such that for all $f \in \ell^p(I)$, $Tf = \sum_{j\in I} f(j)u_j$.*

Proof. (i) $\Rightarrow$ (ii) Suppose that $f, g \in \ell^p(I)$ are disjointly supported. Theorem 1.3 implies that Tf and Tg are disjointly supported. Furthermore, since $T \in \mathfrak{I}(\ell^p(I))$, it follows that $\|T\mathrm{e}_i\| = \|T\|\|\mathrm{e}_i\| = \|T\|$, i.e. $\|T\mathrm{e}_i\|$ is constant for all $i \in I$.

(ii) $\Rightarrow$ (iii) For $f \in \ell^p(I)$, we have $f = \sum_{j\in I} f(j)\mathrm{e}_j$ and so $Tf = \sum_{j\in I} f(j)u_j$, where $u_j = T\mathrm{e}_j$, which by part (ii), its norm is constant. Also, by implication (i) $\Rightarrow$ (ii) of Theorem 2.6, the family $\{u_j;\ j \in I\}$ are disjointly supported.

(iii) $\Rightarrow$ (i) Let $c \geq 0$ be the constant value of $\|u_i\|$ for all $i \in I$. Since the family $\{u_i; i \in I\}$ are disjointly supported, it follows that

$$\begin{aligned} \|Tf\|^p &= \sum_{j\in I} \|f(j)u_j\|^p \\ &= c^p\|f\|^p, \end{aligned}$$

for all $f \in \ell^p(I)$. This implies $T \in \mathfrak{I}(\ell^p(I))$.

□

Remark 2.8. Note that on $\ell^2(I)$ or in general, for any Hilbert space H, a bounded linear operator $T : H \to H$ is an isometry if and only if T preserves inner product, i.e. $\langle Tx, Ty\rangle = \langle x, y\rangle$ for all $x, y \in H$. This shows that $T : H \to H$ is an isometry if and only if there exists two Hilbert basis $\{u_i; i \in I\}$ and $\{v_i; i \in I\}$ of H and a one-to-one map $\theta : I \to I$ with $Tu_i = v_{\theta(i)}$ for all $i \in I$. This result leads to find all the elements of $\mathfrak{I}(H)$.

By Theorems 2.6 and 2.7, it is obvious that $\mathfrak{I}(\ell^p(I)) \subseteq \mathcal{P}_{ds}(\ell^p(I))$. In the following, we give a bounded linear operator $T : \ell^p \to \ell^p$ belongs to $\mathcal{P}_{ds}(\ell^p)$ and obtain sufficient conditions on T to be in $\mathfrak{I}(\ell^p)$.

Example 2.9. Let $a, b \in \mathbb{R}$ and $T : \ell^p \to \ell^p$, for $f = (f_1, f_2, \ldots) \in \ell^p$ be defined by

$$Tf = (\underbrace{af_1, \ldots, af_1}_{n_1}, \underbrace{bf_1, \ldots, bf_1}_{n_2}, \underbrace{af_2, \ldots, af_2}_{n_3}, \underbrace{bf_2, \ldots, bf_2}_{n_4}, \ldots),$$

lies in $\mathcal{P}_{ds}(\ell^p)$, where (n_k) is a bounded sequence in $\mathbb{N}$ (Example 2.5). In order to get sufficient conditions on T to be in $\mathfrak{I}(\ell^p)$, $\|T\mathrm{e}_k\|$ must be constant, independent of k (Theorem 2.7). For this purpose, it is necessary that the value of

$$\begin{aligned}\|T\mathrm{e}_k\| &= \underbrace{|a|^p+\cdots+|a|^p}_{n_{2k-1}}+\underbrace{|b|^p+\cdots+|b|^p}_{n_{2k}}\\ &= n_{2k-1}|a|^p+n_{2k}|b|^p,\end{aligned}$$

be constant, independent of $k\in\mathbb{N}$. Thus the necessary and sufficient condition to $T\in\mathfrak{I}(\ell^p)$ is

$$n_{2r-1}|a|^p+n_{2r}|b|^p=n_{2s-1}|a|^p+n_{2s}|b|^p \quad \text{for all } r,s\in\mathbb{N}.$$

Let $\mathfrak{c}_0(I)$ be the Banach space of all real sequences vanishing at infinity with the supremum norm that is all real function $f:I\to\mathbb{R}$ which for all $\varepsilon>0$ there exists a finite set $E\subset I$ such that $|f(x)|<\varepsilon$ for all x not in E. In the following, we investigate disjoint support property for the bounded linear operators $T:\mathfrak{c}_0(I)\to\mathfrak{c}_0(I)$. Let $\mathcal{P}_{ds}(\mathfrak{c}_0(I))$ denote the set of all bounded linear operators $T:\mathfrak{c}_0(I)\to\mathfrak{c}_0(I)$ which preserve disjoint support property.

Theorem 2.10. *Let $T\in\mathcal{B}(\mathfrak{c}_0(I))$. Then all rows of T are in $\ell^1(I)$, and moreover for any $i\in I$,*

$$\sum_{j\in I}|(T\mathrm{e}_j)(i)|\le\|T\|.$$

Proof. Let $i\in I$ be fixed and $j,n\in I$ be arbitrary. Let E be an arbitrary finite subset of I, put $\delta_j=\operatorname{sgn}(T\mathrm{e}_j)(i)$ and $x_E=\sum_{j\in E}\delta_j\mathrm{e}_j\in\mathfrak{c}_0(I)$. Thus we have $Tx_E=\sum_{j\in E}\delta_jT\mathrm{e}_j$ which implies that

$$\begin{aligned}(Tx_E)(i) &= \sum_{j\in E}\delta_j(T\mathrm{e}_j)(i)\\ &= \sum_{j\in E}|(T\mathrm{e}_j)(i)|\\ &= |(Tx_E)(i)|\le\|T\|.\end{aligned}$$

Since E is an arbitrary finite subset of I, the assertion follows. □

One can obtain Theorem 2.6, for bounded linear operator $T:\mathfrak{c}_0(I)\to\mathfrak{c}_0(I)$ as follows.

Theorem 2.11. *Let $T:\mathfrak{c}_0(I)\to\mathfrak{c}_0(I)$ be a bounded linear operator. Then the following conditions are equivalent.*

(i) $T\in\mathcal{P}_{ds}(\mathfrak{c}_0(I))$.
(ii) *For all distinct $j,j'\in I$, $T\mathrm{e}_jT\mathrm{e}_{j'}=0$.*

(iii) *There exists a uniformly bounded family of disjointly supported* $\{u_j; j \in I\}$ *in* $\mathfrak{c}_0(I)$ *such that*

$$Tf = \sum_{j\in I} f(j)u_j \quad \text{for all } f \in \mathfrak{c}_0(I).$$

Proof. It is similar to the proof of Theorem 2.6. □

Remark 2.12. Theorem 2.11, in the case $I = \mathbb{N}$, is a particular case of Theorem 2 in [10].

3. Conclusion

In Theorems 2.6 and 2.7, we investigate equivalent conditions for $T : \ell^p(I) \to \ell^p(I)$ to be disjointly supported or to be an isometry. We also obtain some of the important properties of isometries on Banach spaces and derive that the constant coefficients of all isometries on $\ell^p(I)$ are in $\mathcal{P}_{ds}(\ell^p(I))$. We proceed with the study of the bounded linear operators $T : \mathfrak{c}_0(I) \to \mathfrak{c}_0(I)$ which are in $\mathcal{P}_{ds}(\mathfrak{c}_0(I))$, and we prove that all rows of the matrix form of such operators belongs to $\ell^1(I)$. We show that Theorem 2.6 satisfies for a bounded linear operator T on $\mathfrak{c}_0(I)$ and obtain Theorem 2.11, it gives some equivalent conditions for $T : \mathfrak{c}_0(I) \to \mathfrak{c}_0(I)$ to preserve disjoint support.

In Theorem 1.3, for an isometry $T : L^p(\mu) \to L^p(\mu)$, where $2 \neq p \in [1, \infty)$, we have $fg = 0$ if and only if $TfTg = 0$. This theorem raises an open problem that "Does this theorem hold for any bounded linear operators defined on any Banach algebra?". That is under what conditions we have two-sided preserving disjoint support property?

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[1] Department of pure Mathematics, Faculty of Mathematical Sciences, University of Shahrekord, P.O.Box 115, Shahrekord, 88186-34141, Iran.

Email address: `eftekhari-n@sku.ac.ir`

[2] Department of pure Mathematics, Faculty of Mathematical Sciences, University of Shahrekord, P.O.Box 115, Shahrekord, 88186-34141, Iran.

Email address: `bayati.ali@sku.ac.ir`